b) lezárás Z_2-vel

$$Z_2 = \frac{U_2}{I_2} = \frac{U(x=0)}{I(x=0)} = \frac{U_2^+(1+r_2)}{\frac{U_2^+}{Z_0}(1-r_2)} = Z_0\,\frac{1+r_2}{1-r_2} \iff \boxed{r_2 = \frac{Z_2 - Z_0}{Z_2 + Z_0}}$$

- ha $\mathrm{Re}\{Z_2\} \geq 0$, akkor $|r_2| \leq 1$

— (előző órán γ/γ és δ/ε az utolsó)

3) Speciális lezárások

A. Illesztett lezárás $Z_2 = Z_0$ esetén $r_2 = 0$ (azaz nincs reflexió)

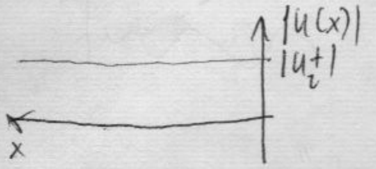
$$U(x) = U_2^+ e^{\gamma x}; \quad \text{vagy } U(z) = U^+ e^{-\gamma z}$$

amplitúdó becslés: $|U(x)| = ?$

most: $|U(x)| = |U_2^+ \cdot e^{\alpha x} \cdot e^{j\beta x}| = |U_2^+| \cdot e^{\alpha x}$

$|\ \ | = 1$

$\alpha = 0$ esetén



B) Nyitott végű ideális TV (szakadással lezárva)

$$Z_2 = \infty \longrightarrow r_2 = \lim_{Z_2 \to \infty} \frac{Z_2 - Z_0}{Z_2 + Z_0} = 1 \longrightarrow$$

totálreflexió / teljes visszaverődés $|r_2| = 1$

$$U_2^- = U_2^+$$

$$U(x) = U_2^+(e^{j\beta x} + e^{-j\beta x}) = 2 \cdot U_2^+ \cos(\beta x);$$

- ha $U_2^+ \in \mathbb{R}$ akkor $u(x,t) = \mathrm{Re}\{2 \cdot U_2^+ \cos\beta x \cdot e^{j\omega t}\} = 2 \cdot U_2^+ \cdot \cos(\beta x) \cdot \cos(\omega t)$

állóhullám alakul ki $\rightarrow$ kioltási helyek v. csomópontok $\quad \lambda = \frac{2\pi}{\beta}$

$$u(x,t) = 0 \ \forall t \iff \cos(\beta x) = 0 \Rightarrow \beta x = \frac{\pi}{2} + k\pi = \frac{\pi}{2}(2k+1) \Rightarrow x = \frac{\pi}{2\beta}(2k+1) = \frac{\lambda}{4}(2k+1)$$

Mi történik az árammal

$$I(x)=\frac{U_2^+}{Z_0}\cdot\left(e^{j\beta x}-e^{-j\beta x}\right)=2j\cdot\frac{U_2^+}{Z_0}\cdot\sin\beta x$$

$$i(x,t)=\mathrm{Re}\left\{\cancel{2}\cdot e^{j\pi/2}\cdot\frac{U_2^+}{Z_0}\cdot\sin\beta x\cdot e^{j\omega t}\right\}=-\frac{2U_2^+}{Z_0}\cdot\sin(\beta x)\cdot\sin(\omega t)$$

ennek zérus helyei $i(x,t)=0\quad\forall t\to\quad\sin\beta x=0$

$$\beta x=k\pi$$

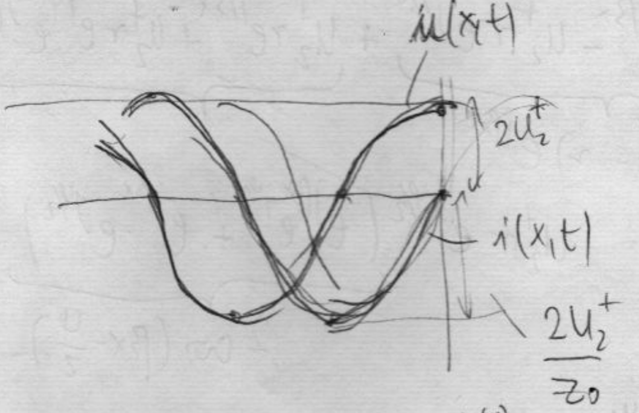
$$x=\frac{k\pi}{\beta}=k\cdot\frac{\lambda}{2}$$

mi van amikor $x=\frac{\lambda}{4}(2k+1)$?

$$x=\frac{\pi}{2\beta}(2k+1)\to i=-\frac{2U_2^+}{Z_0}\cdot\sin(\omega t)\cdot\sin\Big(\underbrace{\frac{\pi}{2\beta}(2k+1)\cdot\beta}_{\frac{\pi}{2}\cdot(2k+1)}\Big)$$

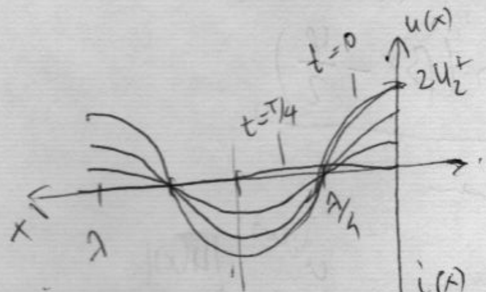
$\frac{\pi}{2}\cdot(2k+1)\Rightarrow$ sin-nek extrémuma van

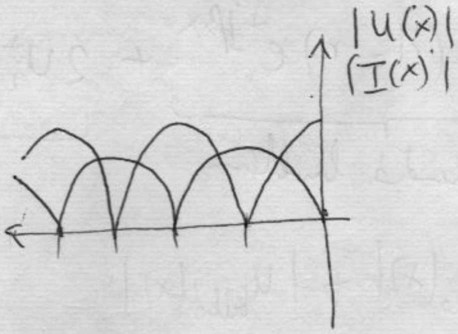
(u-nak ~~max~~ zérusa van

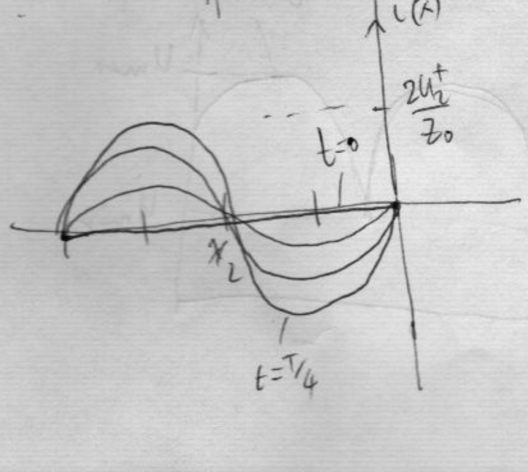


ahol u-nak ~~max~~ duzzadó helye van

ott i-nek maximuma van







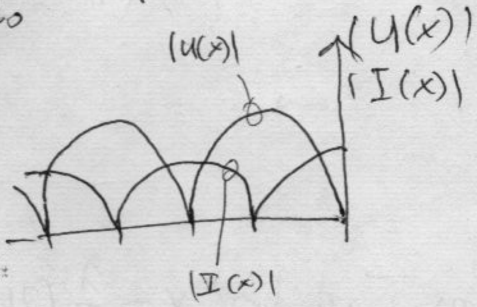
c) Rövidre zárt (ideális TV)

$$\boxed{Z_2 = 0} \longrightarrow r_2 = \frac{Z_2 - Z_0}{Z_2 + Z_0} = -1$$

állóhullámok alakulnak ki

$$\rightarrow U(x) = U_2^+ \left(e^{j\beta x} + (-1)e^{-j\beta x}\right) = 2j \cdot U_2^+ \cdot \sin \beta x$$

$$\rightarrow I(x) = \frac{U_2^+}{Z_0}\left(e^{j\beta x} - (-1)e^{-j\beta x}\right) = 2\,\frac{U_2^+}{Z_0} \cdot \cos \beta x$$



d) Általános (impedancia) leírás

$$\bar{r}_2 = \frac{\bar{Z}_2 - \bar{Z}_0}{\bar{Z}_2 + Z_0} = r_2 \cdot e^{j\varphi}, \quad \text{ahol } r \in \mathbb{R}, \ \varphi \in [0, 2\pi]$$

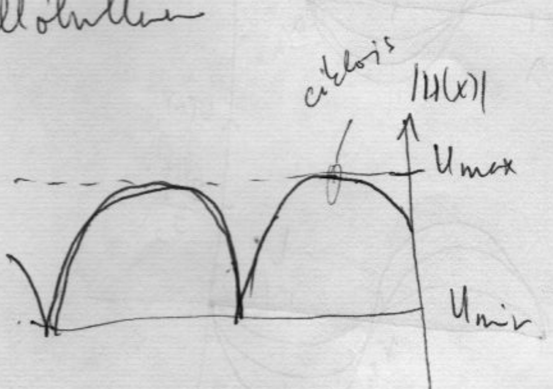
$$U(x) = U_2^+\left(e^{j\beta x} + r e^{j\varphi} e^{-j\beta x}\right) = \underbrace{U_2^+ e^{j\beta x} - U_2^+ r e^{j\beta x}}_{U_2^+(1-r)e^{j\beta x}} + \underbrace{U_2^+ r e^{j\beta x} + U_2^+ r e^{j\varphi} e^{-j\beta x}}$$

$$U_2^+ r e^{j\varphi/2}\underbrace{\left(e^{j\beta x} e^{-j\varphi/2} + e^{-j\beta x} \cdot e^{j\varphi/2}\right)}_{2\cos\left(\beta x - \frac{\varphi}{2}\right)}$$

$$U(x) = \underbrace{U_2^+(1-r)e^{+j\beta x}}_{\text{haladó hullám}} + \underbrace{2U_2^+ \cdot r \cdot e^{j\varphi/2} \cos\left(\beta x - \frac{\varphi}{2}\right)}_{\text{állóhullám}}$$

$$|U(x)| = \underbrace{|U_{\text{haladó}}(x)|}_{U_2^+ \cdot (1-r)} + |U_{\text{álló}}(x)|$$

$$2U_2^+ r \cdot \left|\cos\left(\beta x + \frac{\varphi}{2}\right)\right|$$



5.) Állóhullámarány

a) def. $\boxed{\sigma = \frac{U_{max}}{U_{min}}}$ VSWR – Voltage Standing Wave Ratio

$\sigma \in [1; \infty]$

$\sigma = 1$, ha $U_{max} = U_{min}$ $\Rightarrow$ tisztán haladó

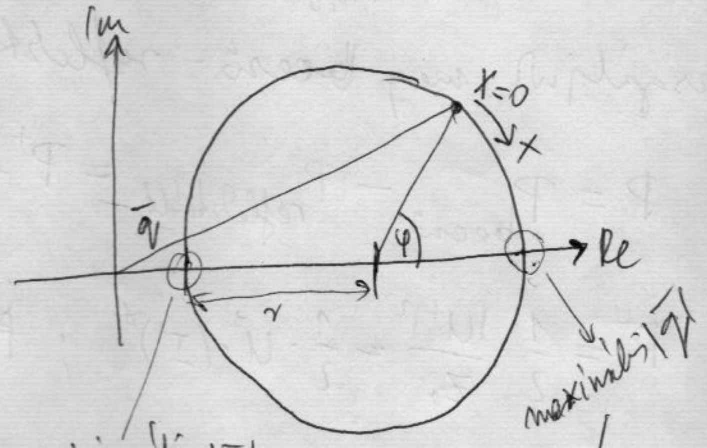
$\sigma \to \infty$, ha $U_{min} \to 0$ $\Rightarrow$ tisztán állóhullám

b) hogyan függ a lezárástól (r_2-n keresztül)

$r_2 = r \cdot e^{j\varphi}$

$$U(x) = U_2^+ \cdot (e^{j\beta x} + r e^{j\varphi} e^{-j\beta x}) = U_2^+ \cdot e^{j\beta x}(1 + r \cdot e^{-j(2\beta x - \varphi)})$$

$$|U(x)| = |U_2^+| \cdot |1| \cdot \underbrace{|1 + r e^{-j(2\beta x - \varphi)}|}_{|\bar{q}|}$$



$U_{max} = \max_x |U(x)| = |U_2^+| \cdot |1 + r|$

$U_{min} = \min_x (|U(x)|) = |U_2^+| \, |1 - r|$

def. alapján

$$\sigma = \frac{U_{max}}{U_{min}} = \frac{|U_2^+| \cdot |1+r|}{|U_2^-| \cdot |1-r|}$$

$$\boxed{\sigma = \frac{1 + |r|}{1 - |r|}}$$

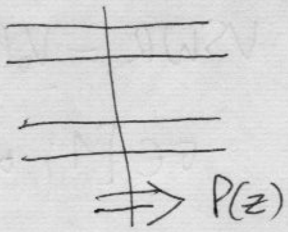
megjegyzés: $U_{max} = |U_2^+| + |U_2^-|$ $\quad (= |U^+| + |U^-|$

$U_{min} = |U_2^+| - |U_2^-|$

Teljesítmény szállítás (lezárt, ideális TV-n)

$$U(z) = U^+ e^{-j\beta z} + U^- e^{j\beta z}$$

$$I(z) = \frac{U^+}{Z_0} e^{-j\beta z} - \frac{U^-}{Z_0} e^{j\beta z}$$



z-nél áthaladó teljesítmény

$$P(z) = \frac{1}{2} \mathrm{Re}\left\{U(z)\cdot I(z)^*\right\} = \frac{1}{2}\mathrm{Re}\left\{ \frac{|U^+|^2}{Z_0^*} \overbrace{- \frac{|U^+||U^-|}{Z_0^*} e^{-j\beta z \cdot 2} + \frac{|U^-||U^+|}{Z_0^*} \cdot e^{j2\beta z}}^{-C+C^* \text{ miatt tisztán képzetes}} + \right.$$

$$\left. - \frac{|U^-|^2}{Z_0^*} \right\} = \frac{1}{2} \cdot \frac{|U^+|^2 - |U^-|^2}{\underset{= Z_0 \text{ mert ideális!}}{Z_0^*}}$$

$P(z)$ a z-től független

vizsgáljuk meg beeső - reflektált szempontból

$$P = P_{beeső} - P_{reflektált} = P^+ - P^-$$

$$P^+ = \frac{1}{2} \frac{|U^+|^2}{Z_0} = \frac{1}{2} \cdot U^+ \cdot (I^+)^* \quad ; \quad P^- = \frac{1}{2} \frac{|U^-|^2}{Z_0} = \frac{1}{2} U^- \cdot (I^-)^*$$

$$P = \frac{1}{2Z_0}\left(|U^+| + |U^-|\right)\left(|U^+| - |U^-|\right) = \frac{1}{2Z_0} \cdot U_{max} \cdot U_{min} = \frac{1}{2Z_0} \cdot \frac{U_{max}^2}{\sigma}$$

$$\mathrm{ch}\,\gamma x = \frac{e^{\gamma x} + e^{-\gamma x}}{2} \;\Rightarrow\; \frac{e^{j\beta x} + e^{-j\beta x}}{2} = \cos(\beta x)$$

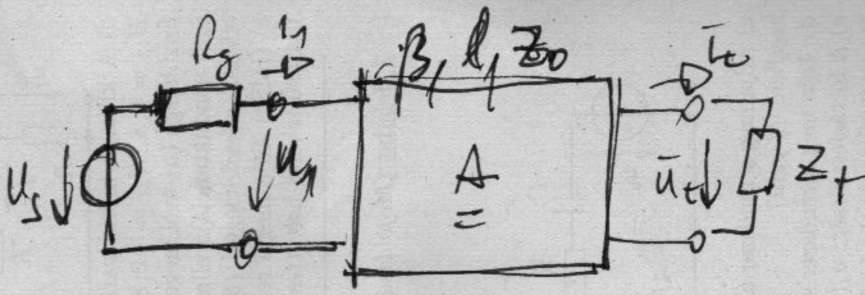
$$\mathrm{sh}\,\gamma x = \frac{e^{\gamma x} - e^{-\gamma x}}{2} \;\overset{\gamma = j\beta}{=}\; \frac{e^{j\beta x} - e^{-j\beta x}}{2} = j \sin(\beta x)$$

γ/bői γ/c után ~~innen fo~~ ideális TV-re innen állandó

Ideális TV

$\gamma = j\beta$

l, Z_0



$$A = \begin{pmatrix} \cos\beta l & jZ_0 \sin\beta l \\ \frac{j}{Z_0}\sin\beta l & \cos\beta l \end{pmatrix}$$

$f = 3\,\text{GHz} \quad \lambda = 0{,}1\ \text{m}$

$$\beta = \frac{\omega}{v} =$$

$c = 2{,}12 \cdot 10^8\ \text{m/s}\ ;\ f = 3\ \text{GHz}\ ;\ \omega = 1{,}885 \cdot 10^{10}\ \frac{\text{rad}}{\text{s}}$

$\lambda = \frac{c}{f} = 0{,}0707\ \text{m} \qquad \beta = 88{,}913\ \text{rad/m}$

$Z_0 = 29{,}27\,\Omega\ ; \quad \underset{(l=0{,}5)}{\underline{A}} = \begin{pmatrix} 0{,}8897 & j13{,}3655 \\ j0{,}0156 & 0{,}8897 \end{pmatrix}$

$\bar{U}_s = 10\,\text{V}$

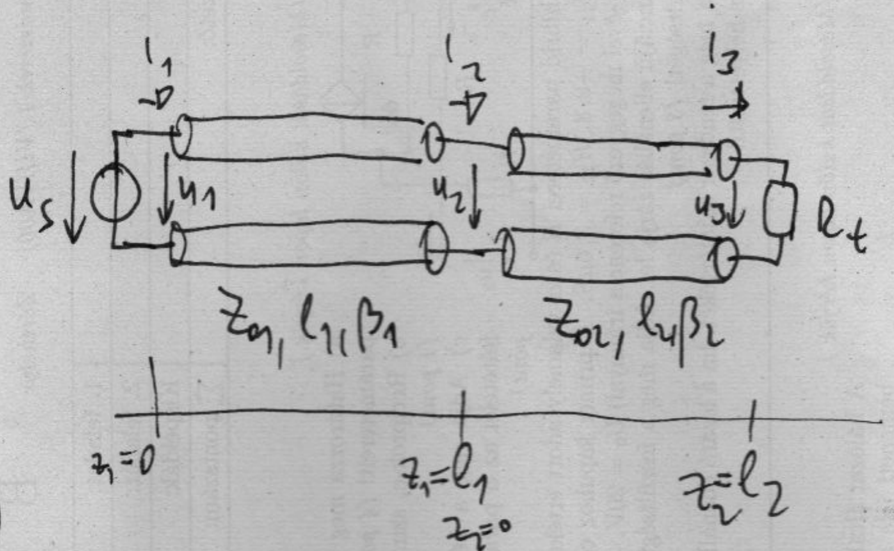
$R_g = 0{,}5\,\Omega$

$Z_t = 5\,\Omega$

$$\left.\begin{aligned} \bar{U}_t &= Z_t \cdot \bar{I}_t \\ \bar{U}_1 &= \bar{U}_s - R_g \cdot I_1 \\ \bar{U}_1 &= A_{11} \cdot \bar{U}_2 + A_{12} \bar{I}_2 \\ I_1 &= A_{21} \bar{U}_2 + A_{22} I_2 \end{aligned}\right\} \quad \begin{array}{cccc} U_1 & I_1 & U_2 & I_2 \end{array}$$

$$\begin{pmatrix} 1 & 0 & -A_{11} & -A_{12} \\ 0 & 1 & -A_{21} & -A_{22} \\ 1 & R_g & 0 & 0 \\ 0 & 0 & 1 & -Z_t \end{pmatrix} \begin{pmatrix} \bar{U}_1 \\ \bar{I}_1 \\ \bar{U}_2 \\ I_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \bar{U}_s \\ 0 \end{pmatrix}$$

$f_{os} = 800\ \mathrm{MHz}$ $\quad c = 3e10^8\ \mathrm{m/s}$ $\quad Z_0 = 50\ \Omega$ $\quad \lambda_g = 0{,}375\ \mathrm{m}$



a)

$$U_1 = A^{(1)}_{11} U_2 + A^{(1)}_{12} I_2$$
$$I_1 = A^{(1)}_{21} U_2 + A^{(1)}_{22} I_2$$
$$U_2 = A^{(2)}_{11} U_3 + A^{(2)}_{12} I_3$$
$$I_2 = A^{(2)}_{21} U_3 + A^{(2)}_{22} I_3$$
$$U_s = U_1 + I_1 \cdot R_g$$
$$U_3 = Z_t \cdot I_3$$

$$\begin{array}{cccccc} U_1 & I_1 & U_2 & I_2 & U_3 & I_3 \end{array}$$
$$\begin{pmatrix} 1 & \cdot & -A^{1}_{11} & -A^{(1)}_{12} & \cdot & \cdot \\ \cdot & 1 & -A^{(1)}_{21} & -A^{(1)}_{22} & \cdot & \cdot \\ \cdot & \cdot & 1 & \cdot & -A^{(2)}_{11} & -A^{(2)}_{12} \\ \cdot & \cdot & \cdot & 1 & -A^{(2)}_{21} & -A^{(2)}_{22} \\ \cdot & - & \cdot & \cdot & 1 & -Z_t \\ 1 & R_g & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \\ \\ \\ \\ \\ \cdot \end{pmatrix} = \begin{pmatrix} \\ \\ \\ \\ \\ U_s \end{pmatrix}$$

U_1^+, U_1^- $\qquad U_2^+, U_2^-$

$$U_1(z_1) = U_1^+ e^{-j\beta z_1} + U_1^- e^{j\beta z_1}$$
$$I_1(z_1) = \frac{U_1^+}{Z_{01}} e^{-j\beta z_1} - \frac{U_1^-}{Z_{01}} e^{j\beta z_1}$$

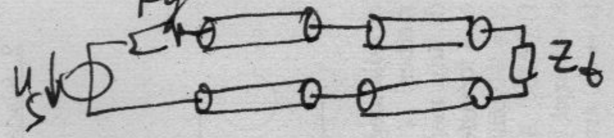
$$U_2(z_2) = U_2^+ e^{-j\beta z_2} + U_2^- e^{j\beta z_2}$$
$$I_2(z_2) = \frac{U_2^+}{Z_{02}} e^{-j\beta z_2} - \frac{U_2^-}{Z_{02}} e^{j\beta z_2}$$

$$U_1 = U_1(0) = U_s - I_1 R_g$$

$$U_1(l_1) = U_2(0)$$
$$I_1(l_1) = I_2(0)$$

$$U_3 = Z_t \cdot I_3$$
$$U_2(l_2) = Z_t \cdot I_2(l_2)$$

① $u_1^+ + u_1^- = u_s - R_g\left(\frac{u_1^+}{Z_{01}} - \frac{u_1^-}{Z_{01}}\right)$



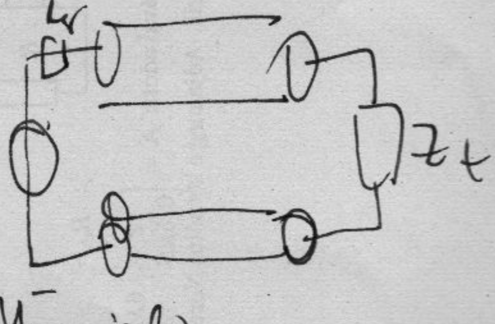
② $u_1^+ e^{-i\beta_1 l_1} + u_1^- e^{i\beta_1 l_1} = u_2^+ + u_2^-$

③ $\frac{u_1^+}{Z_{01}} e^{-i\beta_1 l_1} - \frac{u_1^-}{Z_{01}} e^{-i\beta_1 l_1} = \frac{u_2^+}{Z_{02}} - \frac{u_2^-}{Z_{02}}$

④ $u_2^+ e^{-i\beta_2 l_2} + u_2^- e^{i\beta_2 l_2} = Z_t\left(\frac{u_2^+}{Z_{02}} e^{-i\beta_2 l_2} - \frac{u_2^-}{Z_{02}} e^{i\beta_2 l_2}\right)$

$$\begin{array}{cccc} u_1^+ & u_1^- & u_2^+ & u_2^- \end{array}$$

$$\begin{pmatrix} 1+\frac{R_g}{Z_{01}} & 1-\frac{R_g}{Z_{01}} & \cdot & \cdot \\ e^{-i\beta_1 l_1} & e^{+i\beta_1 l_1} & -1 & -1 \\ \frac{e^{-i\beta_1 l_1}}{Z_{01}} & -\frac{e^{i\beta_1 l_1}}{Z_{01}} & -\frac{1}{Z_{02}} & \frac{1}{Z_{02}} \\ \cdot & \cdot & e^{-i\beta_2 l_2}\left(1-\frac{Z_t}{Z_{02}}\right) & e^{i\beta_2 l_2}\left(1+\frac{Z_t}{Z_{02}}\right) \end{pmatrix} \begin{pmatrix} u_1^+ \\ u_1^- \\ u_2^+ \\ u_2^- \end{pmatrix} = \begin{pmatrix} \cancel{u_1} \\ \cdot \\ \cdot \\ \cdot \end{pmatrix} u_s$$



$u_1^+ + u_1^- = u_s - R_g\left(\frac{u_1^+}{Z_0} - \frac{u_1^-}{Z_0}\right)$

$u_1^+ e^{-i\beta l} + u_1^- e^{i\beta l} = Z_t\left(\frac{u_1^+}{Z_0} e^{-i\beta l} - \frac{u_1^-}{Z_0} e^{i\beta l}\right)$

$$\begin{pmatrix} 1+\frac{R_g}{Z_0} & 1-\frac{R_g}{Z_0} \\ e^{-i\beta l}\left(1-\frac{Z_t}{Z_0}\right) & e^{i\beta l}\left(1+\frac{Z_t}{Z_0}\right) \end{pmatrix} \begin{pmatrix} \\ \\ \end{pmatrix} = \begin{pmatrix} u_s \\ 0 \end{pmatrix}$$