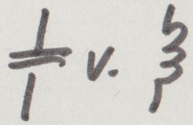
Elsőrendű hálózat

- egyetlen dinamikus elem 

- válasz, belépő gerjesztésre

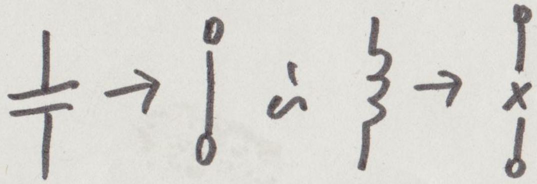
$$y(t) = \varepsilon(t)\cdot\left\{y_g(t) + \left[y(+0) - y_g(0)\right]\cdot e^{-t/\tau}\right\}$$

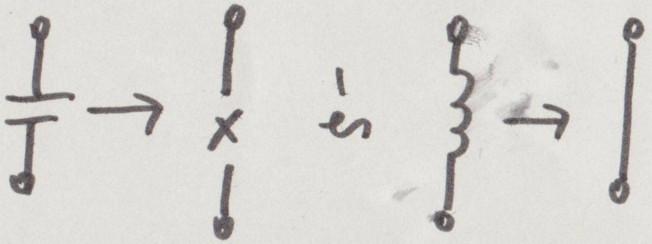
ahol $\tau = C\cdot R_B$ vagy $\tau = \dfrac{L}{R_B}$

R_B: dezaktivizált hálózat bemeneti ell. a dinamikus elem kapcsai felől nézve

ha csatolt kétpólust tartalmaz, akkor célszerűbb az állapotváltozós leírás alapján

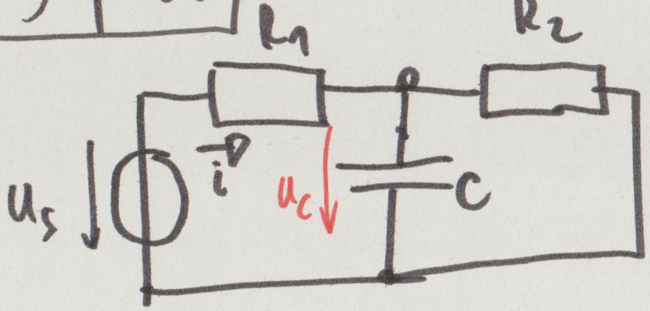
- ha gerjesztés $U_0\,\varepsilon(t)$ vagy $I_0\cdot\varepsilon(t)$

$t=+0$ számítható 

$t\to\infty$ 

helyettesítéssel a hálózat alapján közvetlenül

1.) példa



$R_1 = 20\,\Omega$

$R_2 = 15\,\Omega$

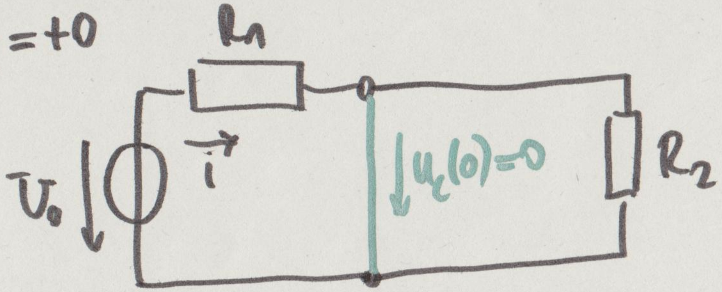
$C = 0{,}5\,\mu F$

$U_0 = 10\,V$

$$u_s(t) = \begin{cases} 0, & t<0 \\ U_0, & t>0 \end{cases}$$

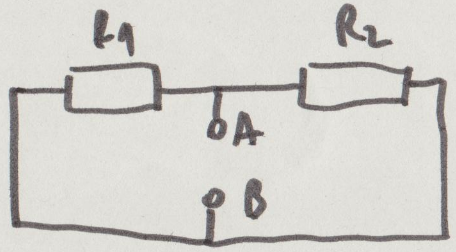
$t<0$ energiamentes

$t = +0$



$$i(+0) = \frac{U_0}{R_1} = \frac{10V}{20\Omega} = 0{,}5\,A$$

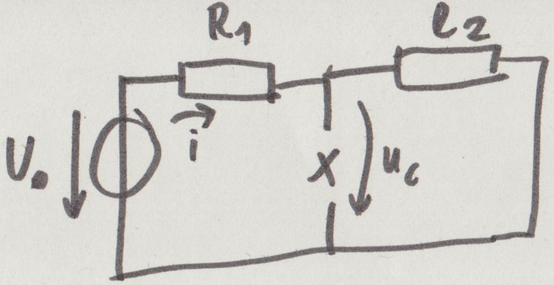
R_B:



$$R_{AB} = R_1 \times R_2 = 20 \times 15 = 8{,}571\,\Omega$$

$$\tau = C \cdot R_B = 0{,}5 \cdot 8{,}571\,\mu s = 4{,}2865\,\mu s$$

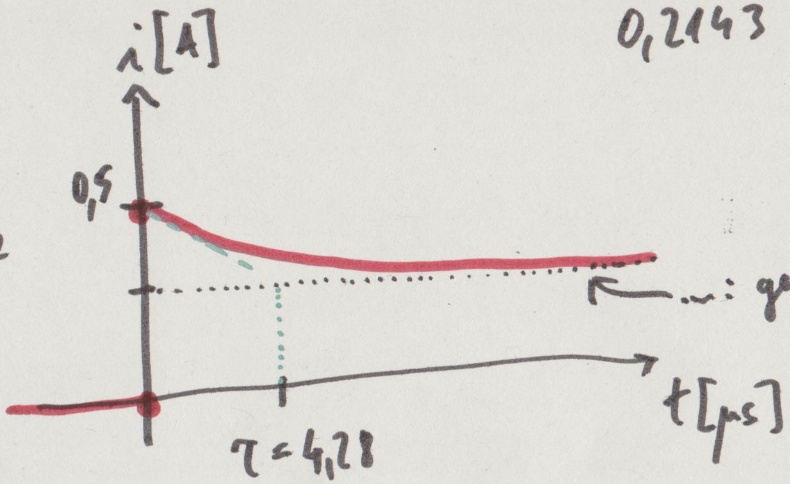
$t \to \infty$:



$$i = \frac{U_0}{R_1+R_2} = \frac{10}{20+15} = 0{,}2857\,A$$

$$u_C = \frac{R_2}{R_1+R_2} \cdot U_0 = \frac{15}{15+20} \cdot 10 = 4{,}2857$$

$$i(t) = \varepsilon(t)\left\{0{,}2857 + \underbrace{(0{,}5-0{,}2857)}_{0{,}2143} \cdot \exp\left(-\frac{t}{4{,}2865}\right)\right\}\ [A]$$

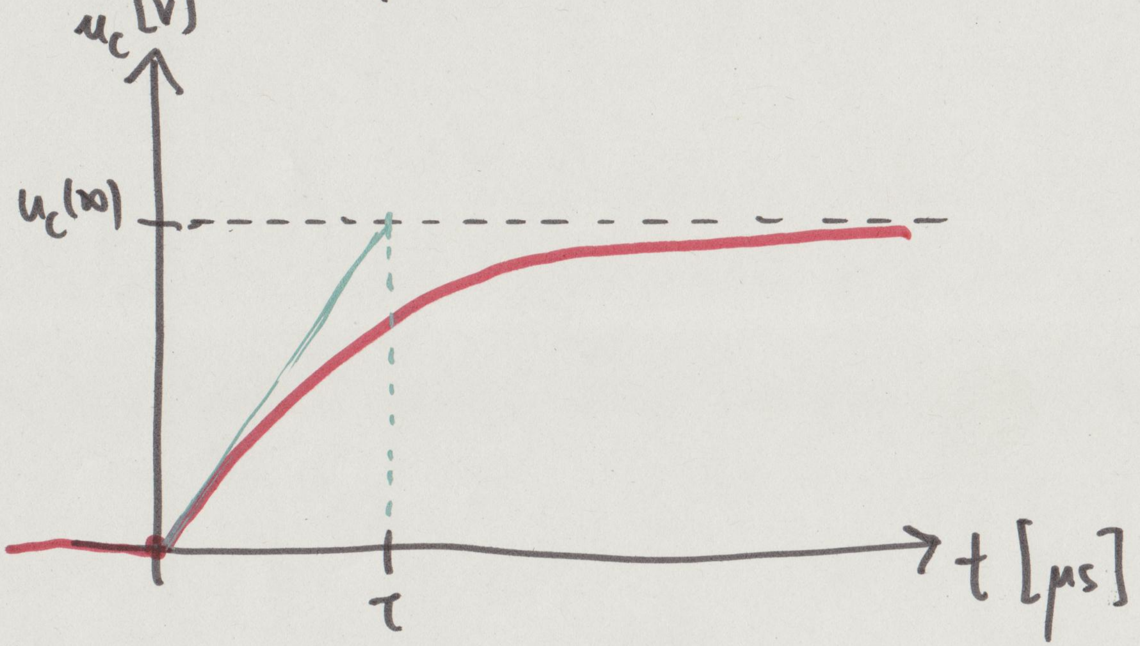


. ugrása van $t=0$-ban

....: gerjesztett válasz

1. példa folytatás

$$u_c(t) = \varepsilon(t)\cdot\left\{4{,}2857 + \underbrace{(0-4{,}2857)}_{-4{,}2857}\cdot\exp\left(-\frac{t}{4{,}2865}\right)\right\}[V] = \varepsilon(t)\cdot 4{,}2857\,V\cdot\left(1-e^{-t/\tau}\right)$$



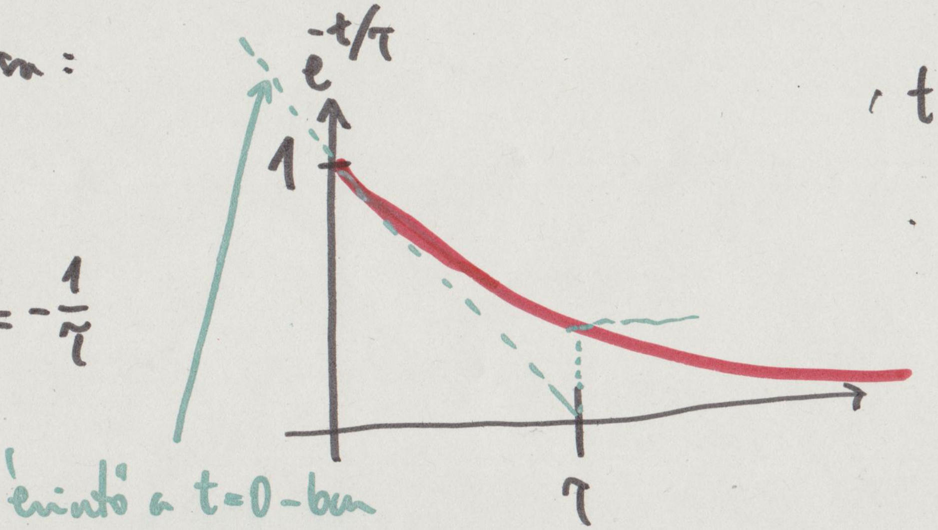
- - - : gerjesztett válasz

(állandósult érték)

Exponenciális fv. rajzolása:

$e^{-t/\tau}$ $t>0$-ra

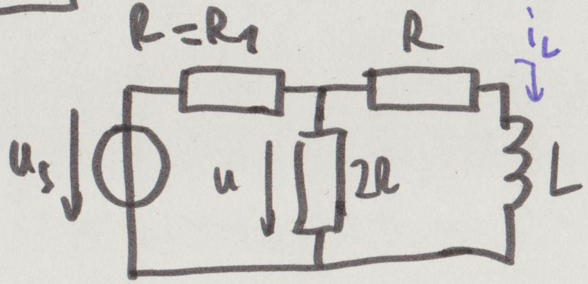
$$\left(e^{-t/\tau}\right)'\Big|_0 = \left(-\frac{1}{\tau}\cdot e^{-t/\tau}\right)\Big|_0 = -\frac{1}{\tau}$$



, $t=\tau$ esetén $e^{-1}=\frac{1}{e}\simeq 0{,}36$

$t=2{,}5\tau$ $e^{-2{,}5}\simeq 0{,}08$

2.



$R = 2\,k\Omega$

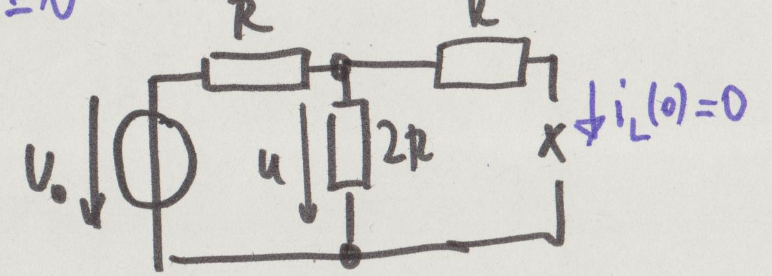
$L = 0{,}1\,\mu H$

$U_0 = 5\,V$

$u_s(t) = U_0 \cdot \varepsilon(t)$

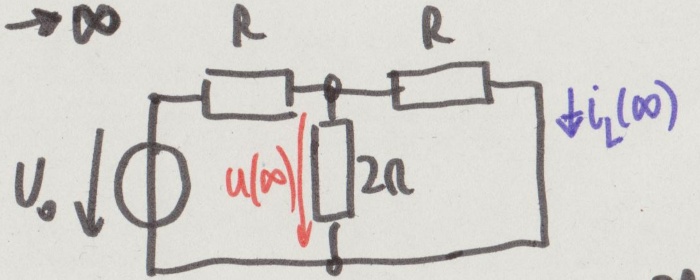
$i_L(0) = 0$

$t = +0$



$$u(+0) = \frac{2R}{R+2R} \cdot U_0 = \frac{2}{3} U_0 = \frac{10}{3}\,V$$

$t \to \infty$



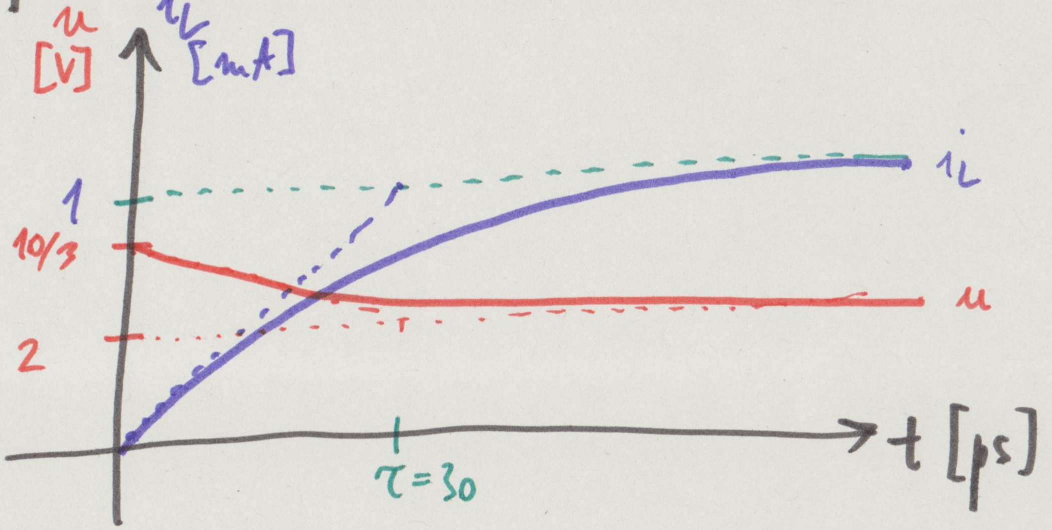
$$i_L(\infty) = \frac{u(\infty)}{R} = \frac{2V}{2k\Omega} = 1\,mA$$

$$u(\infty) = \frac{2R \times R}{R + 2R \times R} U_0 = \frac{2R/3}{5R/3} U_0 = \frac{2U_0}{5} = 2V$$

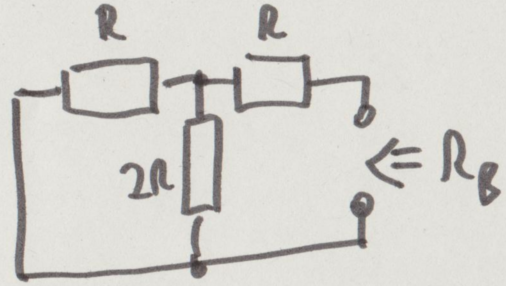
$$i(t) = \varepsilon(t)\left\{1 + (0-1)\, e^{-t/\tau}\right\} = 1\,mA\ \varepsilon(t)\,(1 - e^{-t/\tau})$$

$$u(t) = \varepsilon(t)\left\{2 + \left(\frac{10}{3} - 2\right) e^{-t/\tau}\right\} = \left(2 + \frac{4}{3} e^{-t/\tau}\right)\varepsilon[t]$$

[V]



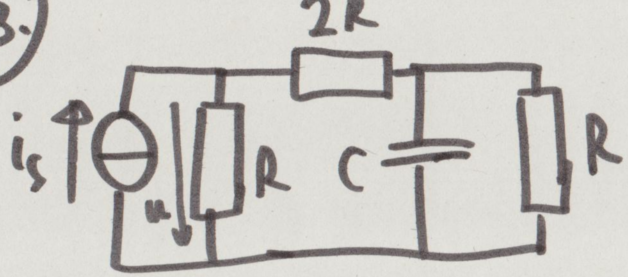
R_B:



$$R_B = R + R \times 2R = \frac{5R}{3} = \frac{10}{3}\,k\Omega$$

$$\tau = L \cdot R_B = 0{,}1 / \frac{10}{3} = 0{,}03\ ns = 30\ ps$$

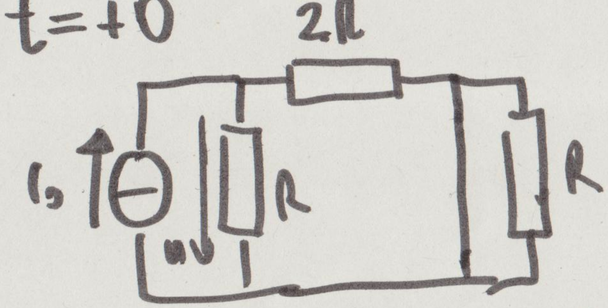
3.)



$i_s(t) = 10\,mA \cdot \varepsilon(t)$

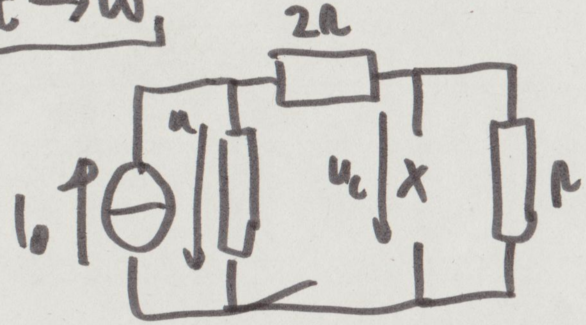
$R = 890\,\Omega;\ C = 148\,nF$

$t = +0$



$u(+0) = I_0 \cdot (R \times 2R) = \frac{2RI_0}{3} = 5{,}933\,V$

$t \to \infty,$



$u(\infty) = I_0 \cdot R \times (3R) = \frac{3RI_0}{4}$

$= 6{,}675\,V$

$u_C(\infty) = \frac{2}{3} \cdot u(\infty) = \frac{RI_0}{2} = 4{,}45\,V$

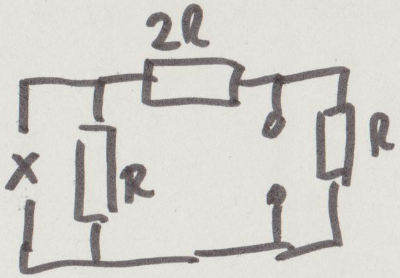
$k\Omega,\ V,\ mA,\ \mu F,\ ms$

$$u(t) = \left(6{,}675 + \underbrace{(5{,}933 - 6{,}675)}_{-0{,}742} e^{-t/98{,}79}\right) \varepsilon(t) \cdot [V]$$

$$u_C(t) = \left(4{,}45 + (0 - 4{,}45)\, e^{-t/98{,}79}\right) \cdot V \cdot \varepsilon(t)$$

$$= 4{,}45 \cdot V \cdot \varepsilon(t) \cdot \left(1 - e^{-t/98{,}79}\right)$$

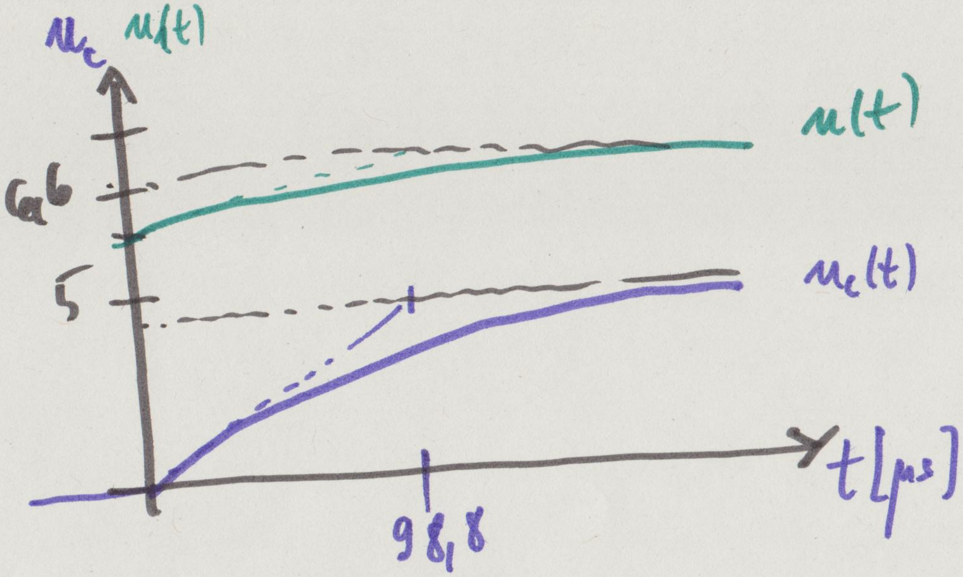
R_B:



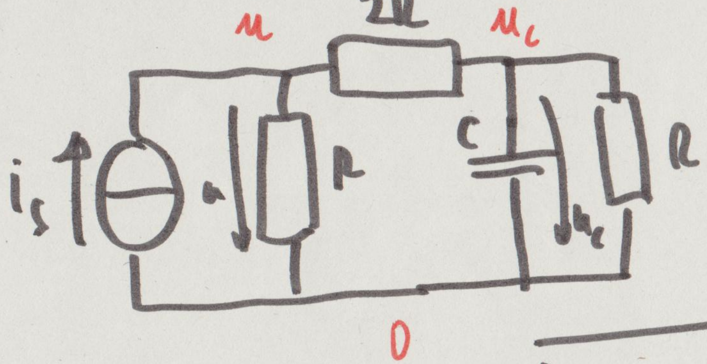
$R_B = R \times (2R + R) = 3R \times R = \frac{3R}{4}$

$\tau = C \cdot R_B = 0{,}09879\,ms$

$= \underline{98{,}79\,\mu s}$



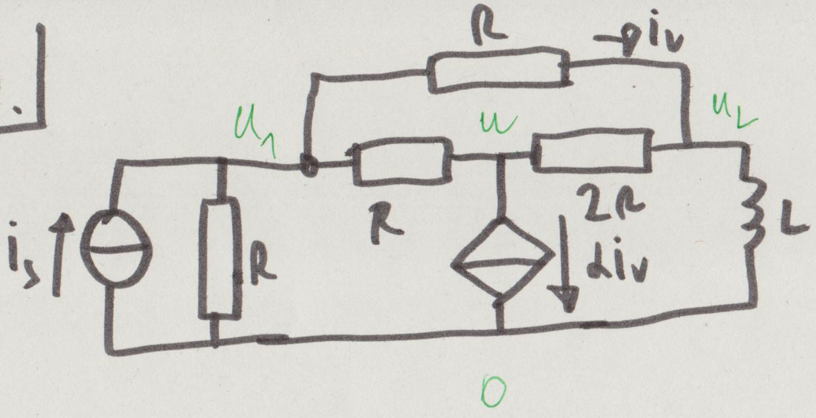
állapotváltozós leírás:



$$\left.\begin{aligned} -i_s + \frac{u}{R} + \frac{u-u_c}{2R} &= 0 \\ Cu_c' + \frac{u_c}{R} + \frac{u_c-u}{2R} &= 0 \end{aligned}\right\}$$

$$\boxed{\begin{aligned} u_c' &= -\frac{4}{3RC}u_c + \frac{1}{3C}i_s \\ u &= \frac{1}{3}u_c + \frac{2R}{3}i_s \end{aligned}}\quad \boxed{\begin{aligned} &= -0{,}01012\, u_c + 0{,}00225\, i_s \\ &= 0{,}333\, u_c + 0{,}5933\, i_s \end{aligned}}$$

4.



vál: u_1

$$i_v = \frac{u_1 - u_L}{R} \quad (1) \qquad u_L = L \cdot i_L' \quad (0)$$

$$i_L + \frac{u_L - u_1}{R} + \frac{u_L - u}{2R} = 0 \quad (2)$$

$$\alpha \cdot i_v + \frac{u - u_L}{2R} + \frac{u - u_1}{R} = 0 \quad (3)$$

$$-i_s + \frac{u_1}{R} + \frac{u_1 - u}{R} + \frac{u_1 - u_L}{R} = \quad (4)$$

$$\Rightarrow \boxed{\begin{aligned} i_L' &= -\frac{(2\alpha+7)R}{(4-\alpha)L}\, i_L + \frac{R}{L} i_s \\ u_1 &= -\frac{2\alpha+4}{4-\alpha} R \cdot i_L + R \cdot i_s \end{aligned}}$$

→ sajátérték:

$$\lambda = -\frac{R}{L}\,\frac{2\alpha+7}{4-\alpha}$$

$\lambda < 0$, ha $\dfrac{2\alpha+7}{4-\alpha} > 0$

a) $2\alpha + 7 > 0 \to \alpha > -\frac{7}{2}$
és
$4 - \alpha > 0 \to \alpha < 4$ vagy

$-7/2$, 4, α

b) $2\alpha + 7 < 0$
$\alpha < -\frac{7}{2}$
és
$4 - \alpha < 0$
$4 < \alpha$ ↯

Stabil, ha $-\frac{7}{2} < \alpha < 4$

$R = 2{,}5\ k\Omega$; $L = 3\ mH$; $\alpha = 0{,}5$; $I_0 = 10\ mA$

koherens egységrendszer: $k\Omega$, mH, V, mA, μs, $Mrad/s$

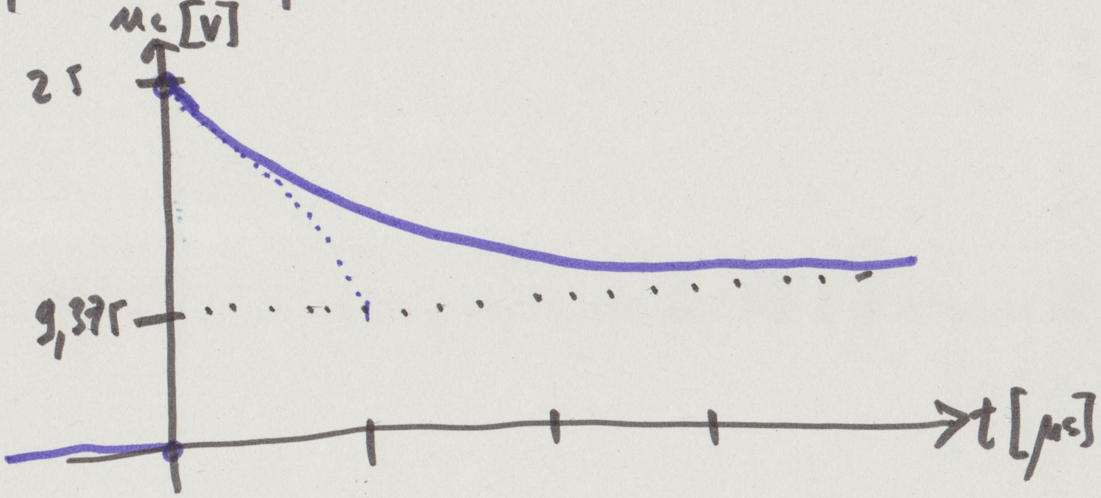
$$\left.\begin{aligned} i_L' &= -\frac{40}{21} i_L + \frac{5}{6} i_s \\ u_1 &= -\frac{25}{7} i_L + \frac{5}{2} i_s \end{aligned}\right\}$$

$$\tau = -\frac{1}{\lambda} = -\frac{1}{-\frac{40}{21}} = \frac{21}{40}\ \mu s = 0{,}525\ \mu s$$

$$u_1(+0) = \frac{5}{2} \cdot 10 = 25\ V \quad (\text{mert } i_L(0) = 0)$$

$$u_1(t) = \left(9{,}375\ V + (25 - 9{,}375)\ V \cdot e^{-t/0{,}525}\right)\varepsilon(t)$$

$$= \left(9{,}375\ V + 15{,}625\ V \cdot e^{-t/0{,}525}\right)\varepsilon(t)$$



$$u_1(\infty) = -\frac{25}{7} i_L(\infty) + \frac{5}{2} I_0$$

$i_L(\infty)$: $t \to \infty$ esetén $i_L' = 0$ (konstans gerjesztés)

$$0 = -\frac{40}{21} i_L(\infty) + \frac{5}{6} I_0$$

$$i_L(\infty) = \frac{5 I_0}{6} : \left(\frac{40}{21}\right) = 4{,}375\ mA$$

$$u_1(\infty) = -\frac{25}{7} \cdot 4{,}375 + \frac{5}{2} \cdot 10 =$$

$$= 9{,}375\ V$$