Másodrendű

Általános

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}' = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} B_1 \\ B_2 \end{pmatrix} u$$

megoldás

$$\underline{x} = \sum_{k=1}^{N} p_k \cdot \underline{m}_k \cdot e^{\lambda_k \cdot t} + \underline{x}_g$$

→ $\underline{m}_k$, λ_k : sajátvektor és sajátérték

→ gerjesztett összetevő, tfh. $u = \begin{cases} 0, & t<0 \\ U_0, & t>0 \end{cases}$

$\underline{x}_g = \begin{pmatrix} Q_1 \\ Q_2 \end{pmatrix}$, ahol Q_1, Q_2 állandó

$$\Rightarrow \quad 0 = \underline{\underline{A}}\,\underline{x}_g + \underline{B} \cdot U_0$$ (← lineáris egyenletrendszer)

$$\underline{\underline{A}}\,\underline{x}_g = -\underline{B} \cdot U_0$$

1.) $\underline{\underline{A}} = \begin{pmatrix}-2 & 0\\ 0 & -3\end{pmatrix}$; $\underline{B} = \begin{pmatrix}1\\ 3\end{pmatrix}$; $\underline{c}^T = (2 \quad 1)$; $D = -2$

$\to \left|\lambda \underline{\underline{E}} - \underline{A}\right| = \begin{vmatrix}\lambda+2 & 0\\ 0 & \lambda+3\end{vmatrix} = (\lambda+2)(\lambda+3) = 0 \qquad \lambda_1 = -2 \; ; \; \lambda_2 = -3$

$\underline{m}_1 = \begin{pmatrix}0\\ 1\end{pmatrix}$; $\underline{m}_2 = \begin{pmatrix}1\\ 0\end{pmatrix}$

$\lambda_1 = -3$; $\lambda_2 = -2$

• $u(t) = U_0 \cdot \varepsilon(t)$ → $x(0) = 0$; → $u \equiv U_0 = \text{áll.} \quad t > 0$; $U_0 = 10$

$\underline{x}_g = \begin{pmatrix}Q_1\\ Q_2\end{pmatrix} \qquad 0 = \underline{A} \cdot \underline{x}_g + \underline{B} \cdot U_0 \to \begin{pmatrix}-2 & 0\\ 0 & -3\end{pmatrix}\begin{pmatrix}Q_1\\ Q_2\end{pmatrix} = (-1) \cdot \begin{pmatrix}1\\ 3\end{pmatrix} \cdot 10$

$Q_1 = {}^{10}/_{+2} = +5$

$x_g = \begin{pmatrix}5\\ 10\end{pmatrix}$

$Q_2 = {}^{30}/_{+3} = +10$

• $\underline{x}(0) = 0$

$\left(P_1 \cdot \underline{m}_1 e^{\lambda_1 t} + P_2 \underline{m}_1 \cdot e^{\lambda_2 t} + \underline{x}_g\right)\Big|_0 = P_1 \cdot \underline{m}_1 + P_2 \underline{m}_2 + \underline{x}_g = 0$

másképpen felírva:

$\underline{m}_1 \cdot P_1 + \underline{m}_2 \cdot P_2 = \left(\underline{m}_1 \vdots \underline{m}_2\right) \cdot \begin{pmatrix}P_1\\ P_2\end{pmatrix} = -\underline{x}_g$

$(\;) \cdot P_1 + (\;) \cdot P_2 = \underbrace{\left((\;)(\;)\right)}_{\underline{\underline{M}}} \cdot \begin{pmatrix}P_1\\ P_2\end{pmatrix} = -\underline{x}_g$

$\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}P_1\\ P_2\end{pmatrix} = -\begin{pmatrix}Q_1\\ Q_2\end{pmatrix} \longrightarrow \begin{pmatrix}P_1\\ P_2\end{pmatrix} = \begin{pmatrix}-10\\ -5\end{pmatrix}$

• teljes megoldás

$x_1 = (-10) \cdot (0) \cdot e^{-3t} + (-5) \cdot (1) e^{-2t} + 5 =$

$= 5 - 5 e^{-2t}$

$x_2 = (-10)(1) \cdot e^{-3t} + (-5) \cdot (0) \cdot e^{-2t} + 10 =$

$= 10 - 10 e^{-3t}$

• $y = (2 \quad 1)\begin{pmatrix}x_1\\ x_2\end{pmatrix} + (-2) \cdot u = 2 \cdot 5 \cdot (1 - e^{-2t}) + 1 \cdot 10 \cdot (1 - e^{-3t}) +$
$+ (-2) \cdot 10 =$

$= -10 e^{-2t} - 10 e^{-3t}$, $\underline{t > 0}$

2.) $\underline{\underline{A}} = \begin{bmatrix} -2 & 1 \\ 0 & -3 \end{bmatrix}$ $\quad B = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$; $C^T = (2 \quad 1)$; $D = -2$; $u(t) = U_0 \cdot \varepsilon(t)$

10

$$|\lambda \underline{\underline{E}} - \underline{\underline{A}}| = \begin{vmatrix} \lambda+2 & -1 \\ 0 & \lambda+3 \end{vmatrix} = (\lambda+2)(\lambda+3) + 1 \cdot 0 = (\lambda+2)(\lambda+3) = 0$$

$$\lambda_1 = -2 \qquad \lambda_2 = -3$$

$$\begin{pmatrix} -2 & 1 \\ 0 & -3 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = (-2) \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$$

$v_2 = 0$

$v_1 = 1$

$\underline{u}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$$\begin{pmatrix} -2 & 1 \\ 0 & -3 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = (-3) \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$$

$v_2 = 1$

$-2v_1 + v_2 = -3v_1 \rightarrow v_1 = -v_2 = -1$

$\underline{u}_2 = \frac{1}{\sqrt{2}} \cdot \begin{pmatrix} -1 \\ 1 \end{pmatrix}$

emlékeztető analízisből

• $\underline{x}_g = \begin{pmatrix} Q_1 \\ Q_2 \end{pmatrix}$ $\quad 0 = \underline{\underline{A}}\,\underline{x}_g + \underline{B}\cdot U_0 \Rightarrow \underline{x}_g = (\underline{\underline{A}}^{-1})\cdot(-\underline{B}\cdot U_0) = \begin{pmatrix} 10 \\ 10 \end{pmatrix}$

• $\underline{x}(0) = 0$ (belépő gerjesztés miatt)

$$0 = \underline{m}_1 \cdot P_1 + \underline{m}_2 \cdot P_2 + \begin{pmatrix} Q_1 \\ Q_2 \end{pmatrix} \rightarrow \underline{\underline{m}}\,\underline{P} = -\underline{x}_g$$

$$\underline{P} = \begin{pmatrix} -20 \\ -10\sqrt{2} \end{pmatrix} \qquad \underline{\underline{m}} = \begin{pmatrix} 1 & -1/\sqrt{2} \\ 0 & 1/\sqrt{2} \end{pmatrix}$$

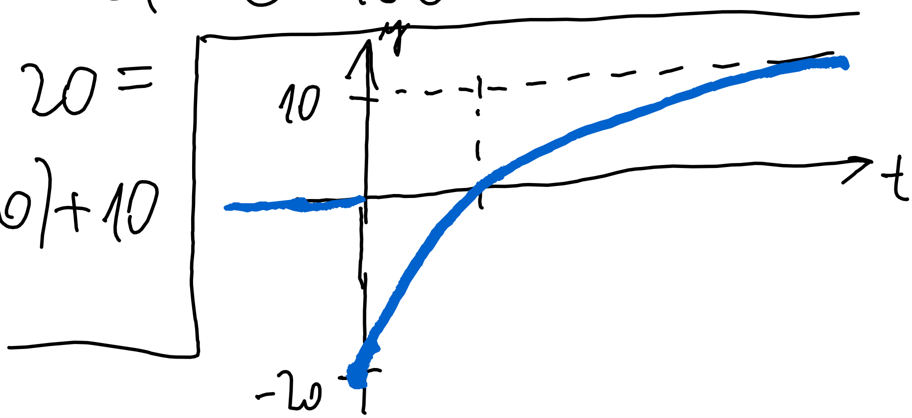
$$x_1 = -20\cdot e^{-2t} \underbrace{-\sqrt{2}\cdot 10\cdot\left(-\frac{1}{\sqrt{2}}\right) e^{-3t}}_{10e^{-3t}} + 10 = -20e^{-2t} + 10e^{-3t} + 10$$

$$x_2 = \frac{1}{\sqrt{2}}\cdot(-10)\sqrt{2}\cdot e^{-3t} + 10 = -10e^{-3t} + 10$$

$$y = 2\cdot x_1 + 1\cdot x_2 - 2\cdot 10 = 2\cdot(-20e^{-2t} + 10e^{-3t} + 10) +$$

$$+ (-10e^{-3t} + 10) - 20 =$$

$$= e^{-2t}(-40) + e^{-3t}(10) + 10$$



3.) $\underline{\underline{A}} = \begin{bmatrix} -2 & 1 \\ 2 & -3 \end{bmatrix} \longrightarrow \lambda_1 = -1; \quad \underline{m}_1 = \frac{\sqrt{2}}{2} \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$\lambda_2 = -4; \quad \underline{m}_2 = \begin{pmatrix} -0{,}4472 \\ 0{,}8944 \end{pmatrix}$

$\underline{B} = \begin{pmatrix} 1 \\ 3 \end{pmatrix};$

$\underline{C}^T = (2 \ \ 1); \ D = -2; \quad u(t) = 10 \cdot \varepsilon(t)$

- $\underline{x}_g = \begin{pmatrix} Q_1 \\ Q_2 \end{pmatrix} \longrightarrow 0 = \underline{\underline{A}}\,\underline{x}_g + \underline{B} \cdot U_0 \longrightarrow \underline{x}_g = \underline{\underline{A}}^{-1} \cdot (-\underline{B} \cdot 10) = \begin{pmatrix} 15 \\ 20 \end{pmatrix}$

- Kezdeti feltétel: $\underline{x}(0) = 0$ (belépő gerjesztés)

$0 = \underline{m}_1 \cdot p_1 + \underline{m}_2 \cdot p_2 + \underline{x}_g \implies \underline{\underline{M}} \cdot \underline{p} = -\underline{x}_g \implies \underline{p} = \begin{pmatrix} -23{,}57 \\ -3{,}727 \end{pmatrix}$

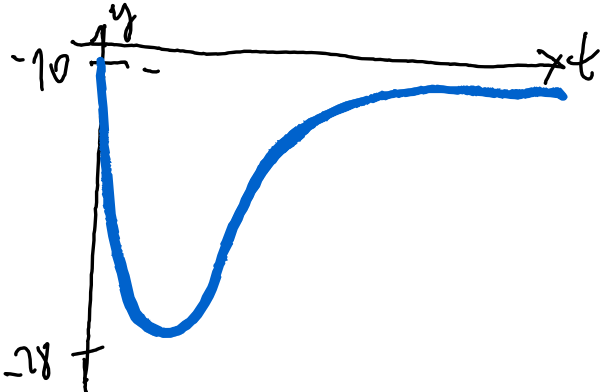
- $x_1(t) = -16{,}66\, e^{-t} + 1{,}66 \cdot e^{-4t} + 15$

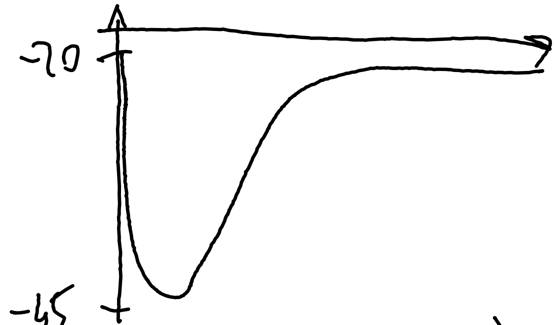
$x_2(t) = -16{,}66 \cdot e^{-t} - 3{,}33 \cdot e^{-4t} + 20$

$y = \underline{C}^T \cdot \underline{x} + D \cdot u = -50 e^{-t} + 50 - 20 = -50 e^{-t} + 30$

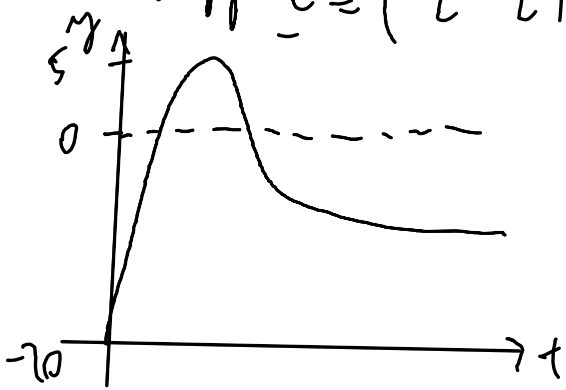
(itt a λ_2 hatása nem jelenik meg a válaszban!)

3/b.) Legyen $\underline{c}^T = [2 \ \ -1]$; vagy $\underline{c}^T = [2 \ \ -2]$





vagy $\underline{c}^T = [-2 \ \ 2]$



Csak a válasz változik, $f_1(t)$ és $f_2(t)$ NEM!

4)