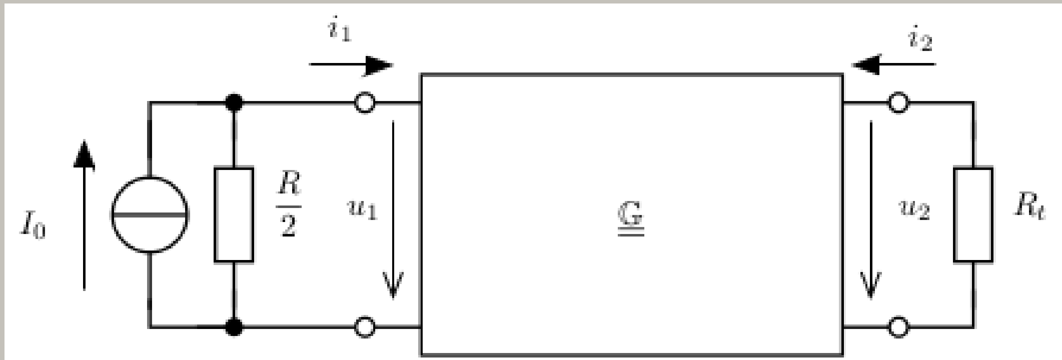
6. gyakorlat – Kétkapu alkalmazása – helyettesítő kapcsolások

1)



$I_0 = 10\,mA$; $R = 4\,k\Omega$

$$\underline{\underline{G}} = \left(\frac{1}{k}\right)\cdot\begin{pmatrix}2 & -1\\ -1 & 3\end{pmatrix}$$

a) $G \to \Pi$ helyettesítő kapcs.

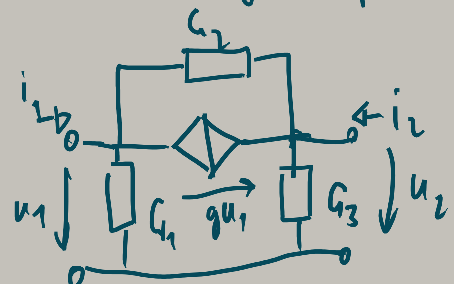
b) $R_t = ?$ $P_{max} = ?$

↳ hely. kép ill. mat. módszer

a) admittancia kar.:

$$\begin{pmatrix}i_1\\ i_2\end{pmatrix} = \underline{\underline{G}}\cdot\begin{pmatrix}u_1\\ u_2\end{pmatrix}$$

Π-hely. kép (sokféle van)



$$-i_1 + G_1\cdot u_1 + g\cdot u_1 + G_2(u_1 - u_2) = 0$$
$$-i_2 - g u_1 + G_3\cdot u_2 + G_2(u_2 - u_1) = 0$$

$$i_1 = G_{11}\, u_1 + G_{12}\cdot u_2$$
$$i_2 = G_{21}\cdot u_1 + G_{22}\cdot u_2$$

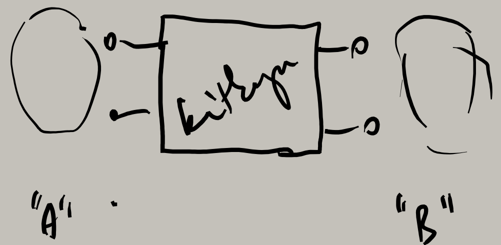
$$\Leftrightarrow \begin{cases} i_1 = (G_1 + g + G_2)\,u_1 - G_2\, u_2 \\ i_2 = -(g + G_2)\,u_1 + (G_2 + G_3)\,u_2 \end{cases}$$

Összevetés alapján:

$$\left.\begin{aligned} G_{11} &= G_1 + G_2 + g \\ G_{12} &= -G_2 \\ G_{21} &= -(g + G_2) \\ G_{22} &= G_2 + G_3 \end{aligned}\right\}
\begin{array}{cccc} G_1 & G_2 & G_3 & g \\ \end{array}$$

$$\begin{pmatrix} 1 & 1 & \cdot & 1 \\ \cdot & -1 & \cdot & \cdot \\ \cdot & -1 & \cdot & -1 \\ \cdot & 1 & 1 & \cdot \end{pmatrix} \begin{pmatrix} G_1 \\ G_2 \\ G_3 \\ g \end{pmatrix} = \begin{pmatrix} G_{11} \\ G_{12} \\ G_{21} \\ G_{22} \end{pmatrix} = \begin{pmatrix} 2/R \\ -1/R \\ -1/R \\ 3/R \end{pmatrix}$$

b)



$\rightarrow$ 4 ismeretlen (u_1, i_1, u_2, i_2)

$\Downarrow$

4 egyenlet kell

2 egyenlet - karakterisztika

1 egyenlet primer oldali leírás

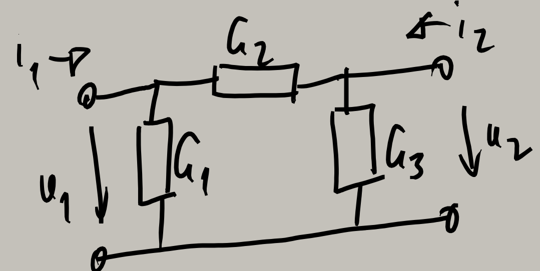
1 —"— szekunder —"—

$G_1 = 0{,}25$ mS $\quad G_3 = 0{,}5$ mS

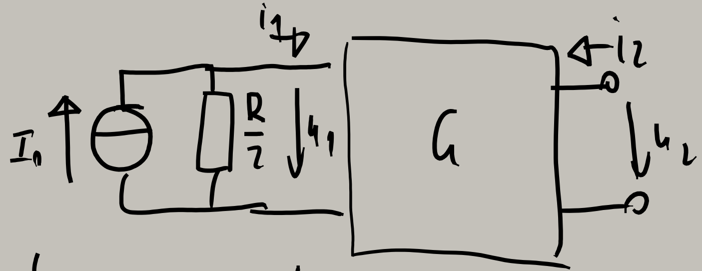
$G_2 = 0{,}25$ mS $\quad g = 0$

$G_{12} = G_{21} \Rightarrow$ reciprok $\Rightarrow$ hely. kép (π-tag)

nem kell vez. forrás



pl. $\underline{\underline{K}} \longrightarrow \underline{\underline{G}} \Rightarrow$ T-tag elemeit



$$-I_0 + \frac{u_1}{R/2} + i_1 = 0 \quad (1)$$

R_t felől számított helyettesítő generátor

$\rightarrow$ $\downarrow u_2 = U_{\ddot{u}}$ $\quad i_2 = 0$ $\quad$ (2)/a

$\rightarrow$ i_{rz} $\downarrow u_2 = 0$ $\quad i_2 = i_{rz}$ $\quad$ (2)/b

(3) $\quad i_1 = \frac{2}{R}u_1 - \frac{1}{R}u_2$

(4) $\quad i_2 = -\frac{1}{R}u_1 + \frac{3}{R}u_2$

$$U_{\ddot{u}} = U_T = 3,6364\,V$$

$$i_{rz} = I_N = -2,5\,mA$$

$$R_B = \frac{U_T}{-I_N} = 1,4545\,k\Omega$$

Üj.

$$\left.\begin{aligned} \frac{2}{R}u_1 + i_1 &= I_0 \\ i_2 &= 0 \\ -\frac{2}{R}u_1 + i_1 + \frac{1}{R}u_2 &= 0 \\ \frac{1}{R}u_1 - \frac{3}{R}u_2 + i_2 &= 0 \end{aligned}\right\}$$

$$\begin{array}{cccc} u_1 & i_1 & u_2 & i_2 \end{array}$$
$$\begin{pmatrix} 2/R & 1 & \cdot & \cdot \\ \cdot & \cdot & \cdot & 1 \\ -2/R & 1 & 1/R & \cdot \\ 1/R & \cdot & -\frac{3}{R} & 1 \end{pmatrix} \begin{pmatrix} u_1 \\ i_1 \\ u_2 \\ i_2 \end{pmatrix} = \begin{pmatrix} I_0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

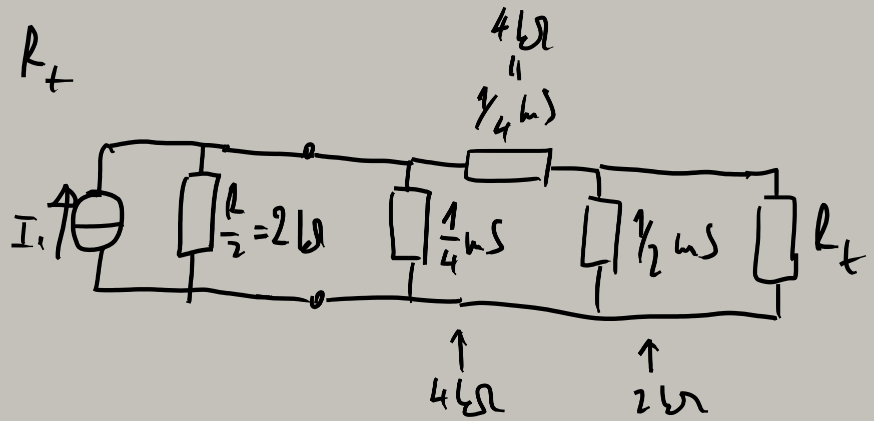
Rz:

$$\left.\begin{aligned} \frac{2}{R}u_1 + i_1 &= I_0 \\ u_2 &= 0 \\ -\frac{2}{R}u_1 + i_1 + \frac{1}{R}u_2 &= 0 \\ \frac{1}{R}u_1 - \frac{3}{R}u_2 + i_2 &= 0 \end{aligned}\right\}$$

$$\begin{pmatrix} 2/R & 1 & \cdot & \cdot \\ \cdot & \cdot & 1 & \cdot \\ -2/R & 1 & 1/R & \cdot \\ 1/R & \cdot & -\frac{3}{R} & 1 \end{pmatrix} \begin{pmatrix} u_1 \\ i_1 \\ u_2 \\ i_2 \end{pmatrix} = \begin{pmatrix} I_0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$R_t = R_B$ $\qquad P_{max} = \frac{U_T^2}{4R_B} = 2{,}2728$ mW

teljesítőleges R_t



fesz. kép alapján

R_t „kívülről" meghat.

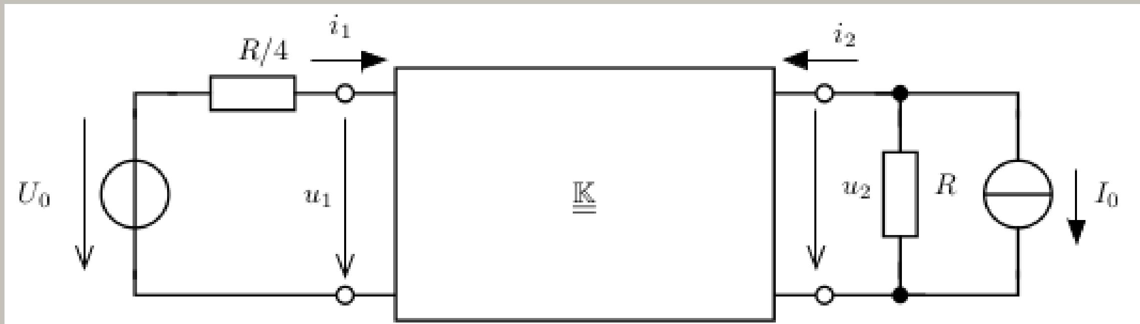
$R_B = ((2 \times 4) + 4) \times 2 = \frac{16}{3} \times 2 = \frac{32/3}{\frac{16}{3}+2} =$

$\frac{4}{3}$

$= \frac{32/3}{\frac{22}{3}} = \frac{32}{22} \cong 1{,}4545$

AMF U_{g1}, I_{r7} a fesz. kép alapján

2.)



$R = 100\,\Omega$; $I = 0{,}2\,A$; $U_0 = 8\,V$

$$\underline{\underline{K}} = \begin{pmatrix} 5/R & 0 \\ -0{,}5 & 2R \end{pmatrix}$$

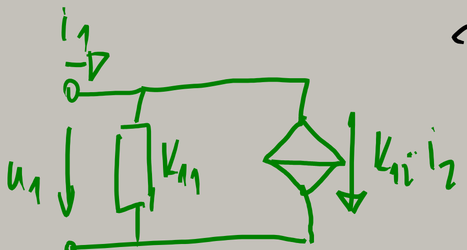
kétkapu fesz és áramok

AHF → egyenlítés 'móka'-val (matematikai megközelítés)

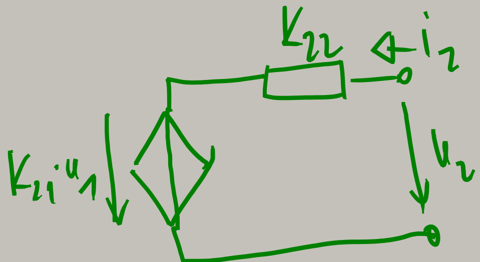
hely. kép: → természetes helyettesítő kép

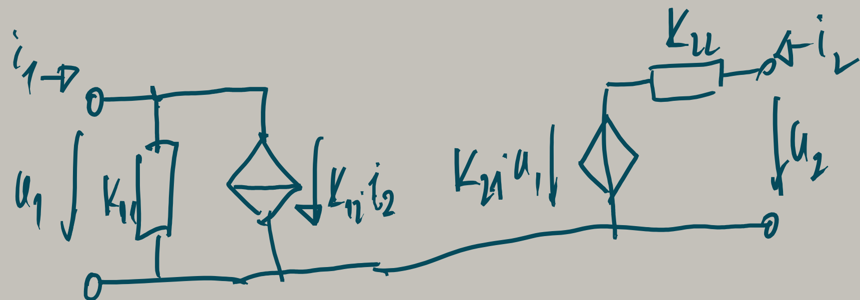
$$\begin{pmatrix} i_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{pmatrix} \begin{pmatrix} u_1 \\ i_2 \end{pmatrix}$$

$$i_1 = \underbrace{K_{11}}_{1/[\Omega]} \cdot u_1 + \underbrace{K_{12}}_{\sim 1} \cdot i_2$$

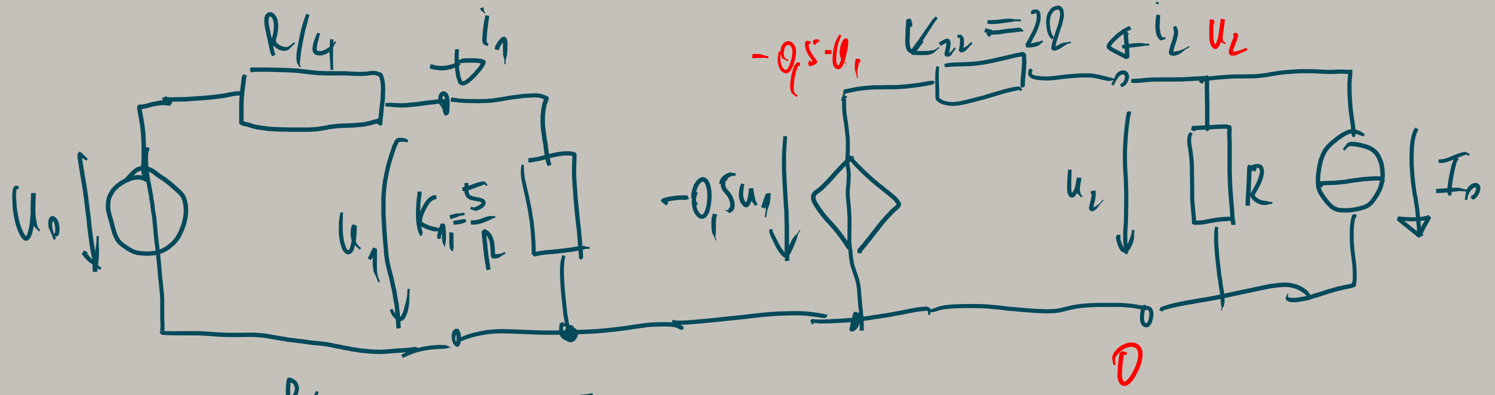


$$u_2 = \underbrace{K_{21}}_{\sim 1} \cdot u_1 + \underbrace{K_{22}}_{[\Omega]} \cdot i_2$$





ha $K_{12} = 0$ →



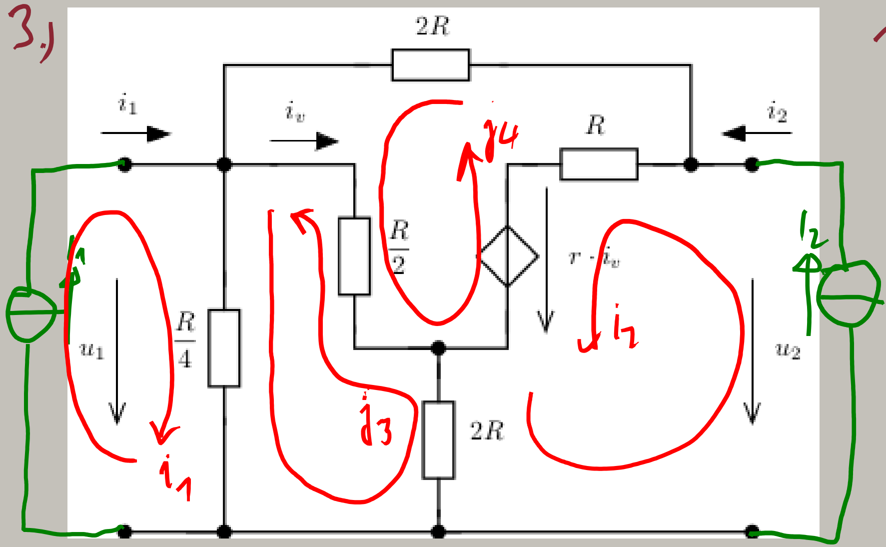
$$u_1 = \frac{R/5}{R/5 + R/4} \cdot U_0 = \frac{5U_0}{9}$$

$$I_1 = \frac{U_0}{\frac{R}{5} + \frac{R}{4}} = \frac{U_0}{\frac{9R}{20}} = \frac{20U_0}{9R}$$

$$\frac{U_2}{R} + \frac{U_2 - (-0{,}5u_1)}{2R} + I_0 = 0 \Rightarrow U_2 = \ldots$$

$$I_2 = \frac{U_2 - (-0{,}5u_1)}{2R} = \ldots$$

3.)



Határozd meg az $\underline{\underline{R}}$ karakterisztikát!

– szimbolikus

– numerikus módon

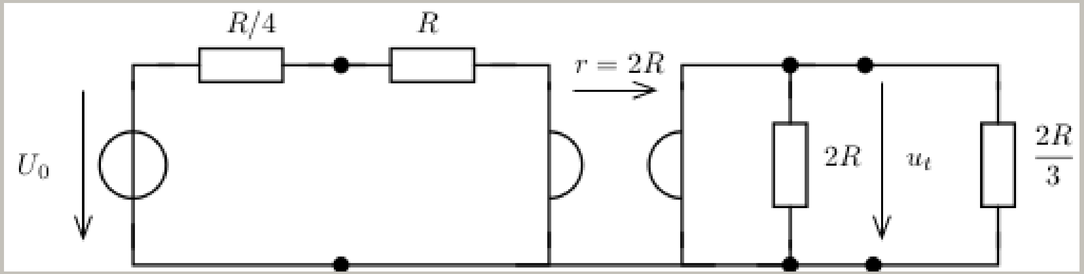
$(r = 0{,}5\,\Omega;\ R = 2\,\Omega)$

u_1, u_2, i_v, j_3, j_4

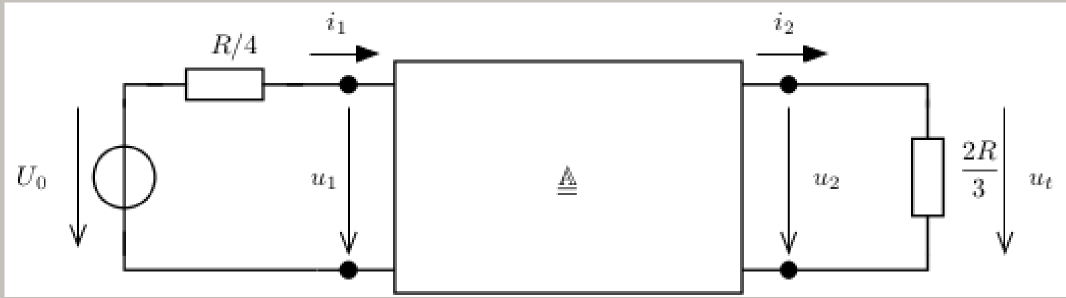
$$\left.\begin{aligned}
i_1&: \ -u_1 + \frac{R}{4}\cdot(i_1 + j_3) = 0\\
i_2&: \ -u_2 + R(i_2 - j_4) + r\cdot i_v + 2R(i_2 - j_3) = 0\\
j_3&: \ \frac{R}{4}\cdot(j_3 + i_1) + 2R(j_3 - i_2) + \frac{R}{2}(j_3 - j_4) = 0\\
j_4&: \ 2R\cdot j_4 + \frac{R}{2}\cdot(j_4 - j_3) - r\cdot i_v + R(j_4 - i_2) = 0\\
&\quad i_v = j_4 - j_3
\end{aligned}\right\}$$

$$\underline{\underline{R}} = \begin{pmatrix} \frac{R(17R-4r)}{3(25R-6r)} & \frac{R(15R-4r)}{3(25R-6r)} \\ \frac{5R^2}{25R-6r} & -\frac{R(-25R+12r)}{25R-6r} \end{pmatrix} = \begin{pmatrix} \frac{64}{141} & \frac{56}{141} \\ \frac{20}{47} & \frac{88}{47} \end{pmatrix}$$

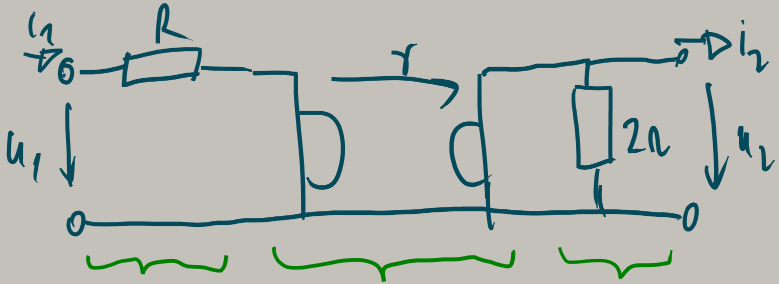
4.



$R = 5\,k\Omega$; $U_0 = 10V$ $u_t = ?$



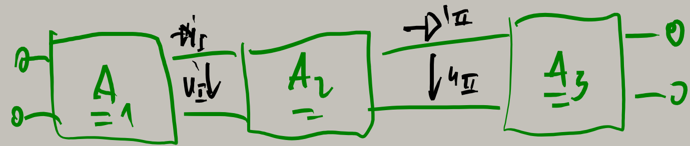
Oldjuk meg az előző feladatot lánckarakterisztikával!



← lánc karakterisztikához tartozó ref. irány

$r = 2R$

$$\begin{pmatrix} u_1 \\ i_1 \end{pmatrix} = \begin{pmatrix} \underline{\underline{A}} \end{pmatrix} \cdot \underbrace{\begin{pmatrix} u_2 \\ i_2 \end{pmatrix}}$$

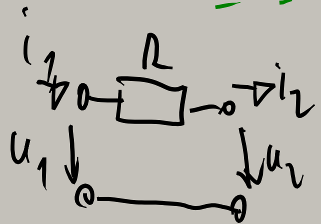


$$\underline{\underline{A}} = \underline{\underline{A}}_1 \cdot \underline{\underline{A}}_2 \cdot \underline{\underline{A}}_3$$

$$\begin{pmatrix} u_1 \\ i_1 \end{pmatrix} = \underline{\underline{A}}_1 \cdot \begin{pmatrix} u_I \\ i_I \end{pmatrix} = \underline{\underline{A}}_1 \cdot \left(\underline{\underline{A}}_2 \cdot \begin{pmatrix} u_{II} \\ i_{II} \end{pmatrix} \right) = \underline{\underline{A}}_1 \left(\underline{\underline{A}}_2 \cdot \left(\underline{\underline{A}}_3 \cdot \underbrace{\begin{pmatrix} u_2 \\ i_2 \end{pmatrix}} \right) \right) =$$

$$= \underline{\underline{A}}_1 \cdot \underline{\underline{A}}_2 \cdot \underline{\underline{A}}_3 \cdot \begin{pmatrix} u_2 \\ i_2 \end{pmatrix}$$

$\underline{\underline{A}}_1$:

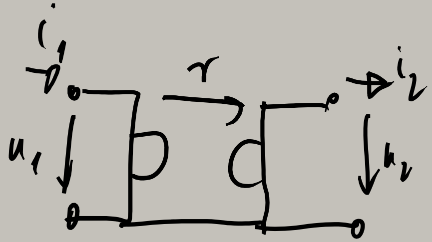


$$\begin{pmatrix} u_1 \\ i_1 \end{pmatrix} = \underline{\underline{A}}_1 \cdot \begin{pmatrix} u_2 \\ i_2 \end{pmatrix}$$

$$\left.\begin{aligned} i_1 &= i_2 \\ u_1 &= u_2 + i_2 R \end{aligned}\right\} \begin{aligned} u_1 &= 1 \cdot u_2 + R \cdot i_2 \\ i_1 &= 0 \cdot u_2 + 1 \cdot i_2 \end{aligned}$$

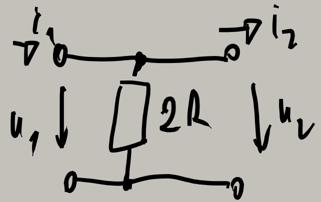
$$\underline{\underline{A}}_1 = \begin{pmatrix} 1 & R \\ 0 & 1 \end{pmatrix}$$

$\underline{\underline{A_1}}$:



$$\left.\begin{aligned} u_1 &= r \cdot i_2 \\ u_2 &= r\, i_1 \end{aligned}\right\} \qquad \begin{aligned} u_1 &= r \cdot i_2 \\ i_1 &= \frac{1}{r} \cdot u_2 \end{aligned} \qquad \underline{\underline{A_2}} = \begin{pmatrix} 0 & r \\ 1/r & 0 \end{pmatrix}$$

$\underline{\underline{A_3}}$:



$$\left.\begin{aligned} u_1 &= u_2 \\ i_1 &= \frac{1}{2\Omega} u_2 + i_2 \end{aligned}\right\} \qquad \underline{\underline{A_3}} = \begin{pmatrix} 1 & 0 \\ 1/2\Omega & 1 \end{pmatrix}$$

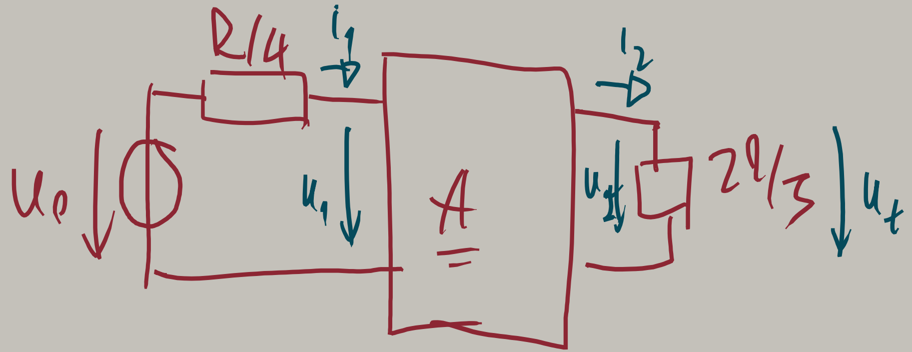
$$\underline{\underline{A}} = \underline{\underline{A_1}} \cdot \underline{\underline{A_2}} \cdot \underline{\underline{A_3}} = \begin{pmatrix} 1 & R \\ 0 & 1 \end{pmatrix} \cdot \underbrace{\begin{pmatrix} 0 & r \\ 1/r & 0 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 1/2\Omega & 1 \end{pmatrix}}_{\begin{pmatrix} r/2\Omega & r \\ 1/r & 0 \end{pmatrix}} = \begin{pmatrix} r/2\Omega + \frac{R}{r} & r \\ 1/r & 0 \end{pmatrix} = \begin{pmatrix} \frac{r^2 + 2R^2}{2\Omega r} & r \\ 1/r & 0 \end{pmatrix}$$

$$r = 2\Omega$$

$$\underline{\underline{A}} = \begin{pmatrix} 3/2 & 2\Omega \\ \frac{1}{2\Omega} & 0 \end{pmatrix}$$

$$\frac{r^2 + 2\Omega^2}{2\Omega r} = \frac{(2\Omega)^2 + 2\Omega^2}{2R \cdot 2\Omega} = \frac{6R^2}{4R^2} = \frac{3}{2}$$

- [illegible]



$$\left.\begin{array}{ll} 1 & u_1 = U_0 - i_1 \cdot (R/4) \\ 2 & u_2 = i_2 \cdot \frac{2R}{3} \\ 3 & u_1 = \frac{3}{2} u_2 + 2R \cdot i_2 \\ 4 & i_1 = \frac{1}{2R} u_2 \end{array}\right\}$$

$$\boxed{U_t = U_2} \Leftarrow \begin{array}{cccc} u_1 & i_1 & u_2 & i_2 \end{array}$$

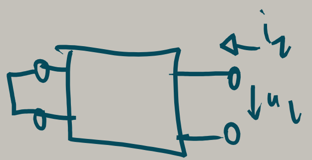
$$\begin{pmatrix} 1 & R/4 & \cdot & \cdot \\ \cdot & \cdot & 1 & -\frac{2R}{3} \\ 1 & \cdot & -3/2 & -2R \\ \cdot & 1 & -1/2R & \cdot \end{pmatrix} \begin{pmatrix} u_1 \\ i_1 \\ u_2 \\ i_2 \end{pmatrix} = \begin{pmatrix} U_0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\underline{\underline{K}} \rightarrow \underline{\underline{G}}$$

$$\left.\begin{aligned} i_1 &= K_{11} \cdot u_1 + K_{12} \cdot i_2 \\ u_2 &= K_{21} \cdot u_1 + K_{22} \cdot i_2 \end{aligned}\right\} \quad \begin{aligned} i_1 &= \dots \\ i_2 &= \dots \end{aligned}$$

$$\left.\begin{aligned} i_1 &= \frac{K_{11} K_{22} - K_{12} K_{21}}{K_{22}} u_1 + \frac{K_{12}}{K_{22}} \cdot u_2 \\ i_2 &= -\frac{K_{21}}{K_{22}} \cdot u_1 + \frac{1}{K_{22}} u_2 \end{aligned}\right\}$$

ha $K_{22} = 0 \Rightarrow \underline{\underline{G}}$ nem értelmezett



reciprocitás $\dfrac{K_{12}}{K_{22}} = -\dfrac{K_{21}}{K_{22}}$

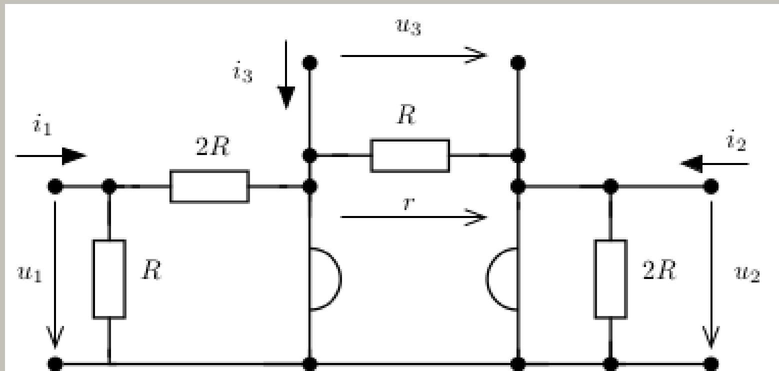
$$\boxed{K_{12} = -K_{21}}$$

„C"

Háromkapu ⇒ A, B, C

hibrid jellegű

„A"

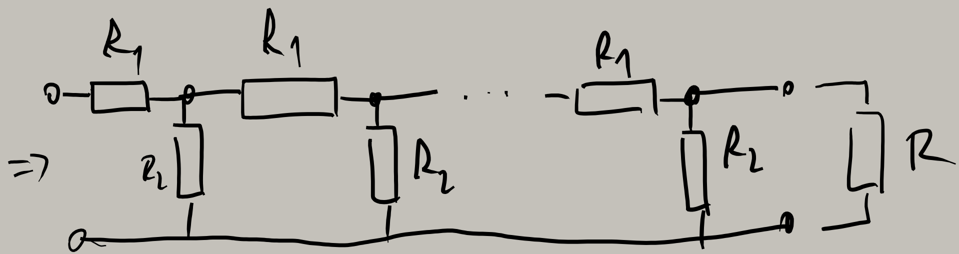


„B"

egyenletek

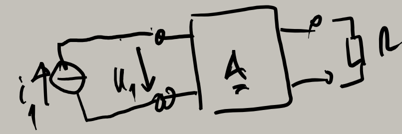
⇒ $\underline{\underline{P}}\,\underline{x} = \underline{\underline{Q}}\,\underline{s} \rightarrow \; = \underline{\underline{P}}^{-1}\underline{\underline{Q}}$

Keresett háromkapu karakterisztika



$\underline{\underline{A}}_0 \qquad \underline{\underline{A}}_0 \qquad \ldots \qquad \underline{\underline{A}}_0$

1. $\qquad$ 2. $\qquad\qquad$ n.



$$\underline{\underline{A}} = (\underline{\underline{A}}_0)^n$$

$$\underline{\underline{A}}_0 \Rightarrow \underline{u}_1 \;\; \underline{u}_2 \qquad \lambda_1, \lambda_2$$

$$\underline{\underline{O}}^T \cdot \underline{\underline{A}}_0 \, \underline{\underline{O}} = \begin{pmatrix} \lambda_1 & \\ & \lambda_2 \end{pmatrix} \qquad (\Lambda)^n = \begin{pmatrix} \lambda_1^n & 0 \\ 0 & \lambda_2^n \end{pmatrix}$$

$$(\qquad)^n = \underline{\underline{O}}^T \underline{\underline{A}}_0 \underbrace{\underline{\underline{O}}\;\underline{\underline{O}}^T}_{\underline{\underline{E}}} \underline{\underline{A}}_0 \underline{\underline{O}} \ldots = \underline{\underline{O}}^T \underline{\underline{A}}_0^n \underline{\underline{O}}$$

$$\underline{\underline{A}}_0^n = \underline{\underline{O}} \cdot \underline{\underline{\Lambda}}^n \cdot \underline{\underline{O}}^T$$