Optimization of particle trajectories inside an ion-thruster

Andras Reichardt* and Arpad L. Makara*

*Department of Broadband Infocommunication and Eletromagnetic Theory,
BUTE, Egry Jozsef u. 18, H-1111 Budapest
e-mail: reichardt.andras@vik.bme.hu

***Abstract*—We present optimization of trajectories of particles that fly from vacuum chamber to the outer world. Expected trajectories are defined by control point regions that must be entered. We present a simple electrostatic model for the system of electrodes. Potentials are calculated using a supervised machine learning (SML) algorithm.**

***Index Terms*—electrostatic approximation, ion thruster, optimization, particle trajectory**

I. Introduction and Motivation

First we introduce our motivation and background used in that paper. Here we present our results about optimization of ion trajectories inside an ion thruster's nozzle region to be able to control very small satellites. In the recent years a lot of small satellites are already launched or planned to launch designed and manufactured at our department (MaSat-1, SMOG-P[1], ATL-1 [2]).

After launching a small satellite and placing it into its final orbit, it can have unwanted and uncontrolled rotation. Therefore, a small satellite needs to perform some maneuvers to stabilize or slightly change its orbit position. Due to its small size, it needs only a small drive system and torque to stabilize. Furthermore, the available energy for attitude control is also limited. In this paper, we focus on tiny satellites with a size of 2-3 U in terms of PocketCube standards. Satellites' small size and mass involves that relatively small force is enough to control such a vehicle compared to thrust needed to put them on to orbit[3], [4].

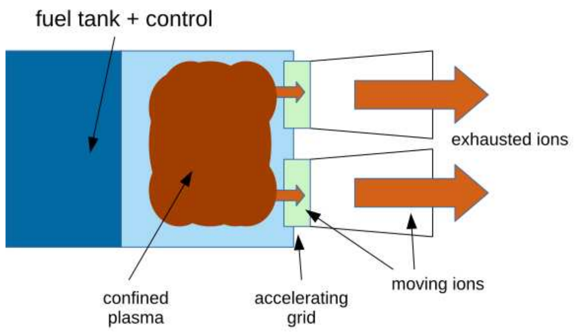

Fig. 1. Schematic of an ion thruster engine

A possible way to steer and stabilize a tiny satellite is using an ion thruster. An ion-thruster is an accelerator that uses Xenon or Iodine to produce positively or negatively charged ions[5], [6]. Ions generated in an ion-chamber using electric field. Xenon is stored in the fuel tank (shown on Fig. 1). Generated ions are accelerated using a pair electrostatic grids called accelerator grid ot fly out to the nozzle of the thruster. Nozzles surface can be used to control electrically the ion beams that flows out from the nozzle. If we control ion beams we can steer the satellite.

In an earlier paper [7] we showed that using a simple pair of electrodes control of beam trajectories can be achieved[7]. The electrodes are on the inner surface of nozzle and its potential can be varied relatively to the potential of AG. During launch of satellite the nozzles are packed up and only after the satellite is set on orbit are unwraped.

Optimization are needed to set ion beams to a prescibred path that are defined using points and weights. These points has to be touched as much as possible flying from first point (start point) toward the last point.

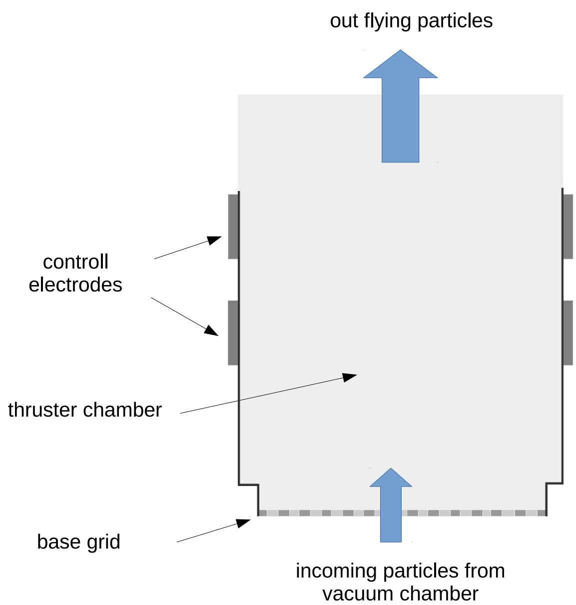

Fig. 2. Outline of the nozzle, wall of nozzle is non-conducting

The paper is organized as follows. In Section II. we propose our model of the electromagnetic problem. Thereafter in Section III we show optimization and supervized machine learning method we used. Results are shown for some cases in Section IV and a summary is given at the end in SectionV.

II. Electromagnetic model of the direct problem

Ions that enter nozzle have a high longitudinal velocity. Therefore the electromagnetic field created by control electrodes can be modeled as a constant field with very slow change in time[8]. In time that ion flies through nozzle the field do not change. Effect of ions to each other neglected just like effect of external magnetic fields. Geometry is shown on Fig. 3 where base electrode is the upper side of the acceleration grid (AG), electrodes are the control electrodes with variable potential. Simulation region is a great volume around nozzle to avoid side effects.

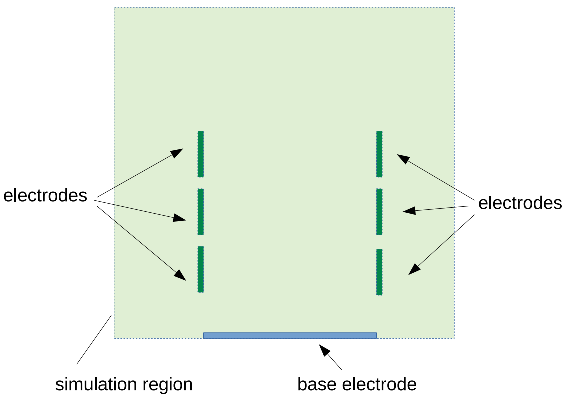

Fig. 3. Geometry of simulated rectangular shaped nozzle

The model that describes electromagnetics of the physical system is an electrostatic model[9]. Potential of the control electrodes (these electrodes are on the surface of nozzle, dark green rectangles on Fig. 3) and potential of base electrode (top electrode of the accelerator grid between plasma chamber and nozzle, blue region on Fig. 3) are specified.

Our goal is to solve this electrostatic model using FEM to calculate potential inside the nozzle. As the potential is known, any trajectory of a charged particle can be calculated[10].

In the simulation region we solve Laplace-eqation (1) with boundary conditions on base electrode ($\varphi = V_0$) and control electrodes ($\varphi = V_i$).

$$\nabla \varepsilon \nabla \varphi = 0 \tag{1}$$

On the border of simulation region homogeneous Neumann condition were used. After solving this model we can calculate any trajectory of ions that start from base electrode solving equation of motion (2) with force calculated from potential (3), where

$$m_{\text{ion}} \cdot \frac{\mathrm{d}^2 \mathbf{r}}{\mathrm{d}t^2} = \mathbf{F} \tag{2}$$

$$\mathbf{F} = (-1) \cdot q_{\text{ion}} \cdot \nabla \varphi \tag{3}$$

Geometrical parameters of nozzle as shown on Fig. 2 are determined by size limits of the satellite and fixed in

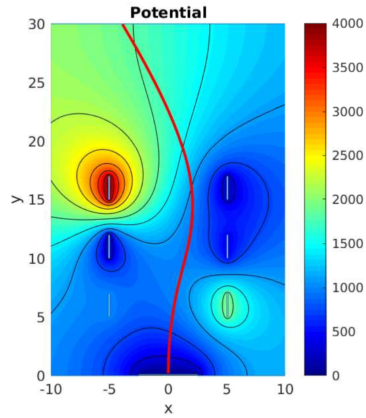

Fig. 4. Electrostatic potential simulated (colored contour) and particle trajectory (red line)

simulations. All supporting structures of the electrodes are made of thin insulator rods and in a normal operation ions do not collide them.

III. Some words about optimization and machine learning

Nowadays, Artificial Intelligence (AI) is a popular buzzword, and many efforts are made to use everywhere. There are specialized computers that offer built-in support for AI-like problems. Our goal using AI was to formulate and prototype this problem from the AI viewpoint. Using these special computers, the calculation of control voltages for satellite trajectory correction could be calculated in real-time.

A. Definition of optimal trajectory

Goal of optimization was to determine voltages of control electrodes to achieve that ionbeams that flows out of nozzle follows a prescribed path inside the nozzle. Path of ionbeams are defined through definition of points and weights along the required path (see Fig. 6).

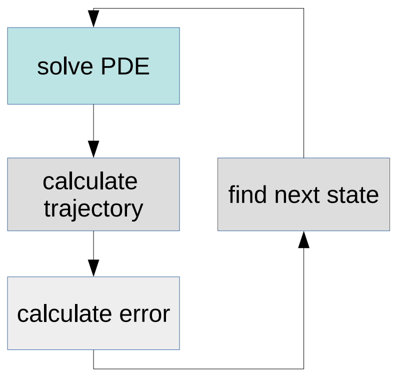

Fig. 5. Schmetic flow chart of the optimization process. Red line indicates trajectory of ion that flies out of acceleration grid. Color contour shows contour plot of electrostatic potential.

Start point of all ions is on the outer surface of acceleration grid (AG). Final point is the point where ion leaves nozzle. Inside points (control points, CPs) are defined too. A possible configuration is shown on Fig. 6.

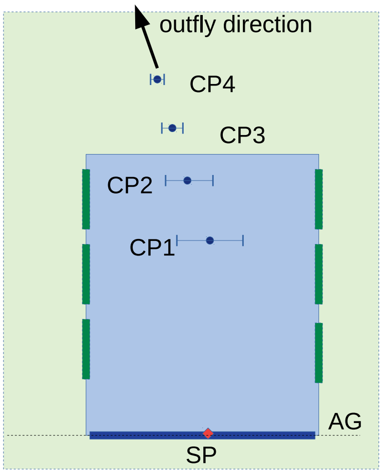

Fig. 6. Control points defined inside and outside of nozzle. Electrodes (green rectangles), top of acceleration grid (AG, blue) and starting point (SP, red) shown. Width of control points are inversely proportional to the weight of them. Wide point means low weight. Light blue region means "geometrical" definition of the nozzle.

Error function is based on deviation of trajectory from control points. Ions have a high velocity perpendicular to the plane of AG's surface it is leaved. Derivation of ion trajectory from CP is calculated with a horizontal distance.

At the k$^{\text{th}}$ CP the error-function is $|x_k - x_{k,pre}|$, where x_k is horizontal coordinate of ion and $x_{k,pre}$ is horizontal coordinate of the control point. Other error-functions can be used in case of other problems, but in this case one used was the best.

Total error-function is calculated as follows :

$$\langle \text{error} \rangle = \sum_{k=1}^{N} p_k \cdot |x_k - x_{k,pre}| \tag{4}$$

where p_k is weigth of kth CP. Using a weights some CPs can be defined as more important than others.

B. Optimization as a machine learning problem

There are a lot of different techniques in machine learning (ML)[12], [13]. We have chosen supervised machine learning (SML) to solve this problem. SML differs from "simple" ML to have a goal-function (or error-function) defined and used to characterize the state.

SML is an iterative process, as shown in Fig. 5. We define a $\mathbf{W}$ parameter vector that contains all the potentials of electrodes. In every step a guess is made using (5) and next $\mathbf{W}$ is chosen. Error is calculated using (4). Iteration stops if the maximum number of iterations reached or the error is less than the limit.

$$\mathbf{W_{next}} = \mathbf{W_{prev}} + 2 \cdot \mu \cdot \langle \text{error} \rangle \cdot p \tag{5}$$

where $\mathbf{W}$ is parameter-vector, μ learning-factor, $\langle \text{error} \rangle$ is sum of errors at all control points, p is scaling factor.

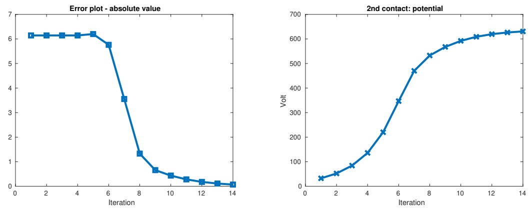

Fig. 7. Errors calculated (measured) after every steps of optimization on the left. Voltage of the 2nd contact is shown on the right in case of the same optimization process.

Learning-factor should be less than $1/2$. Its value is important because if it is small, then convergence is slow, or a "solution" easier can be stucked in a shallow local minimum. A high learning factor can cause large steps in convergence, but sometimes it can cause non-convergence of the process.

IV. Results of Optimization Work

We analyzed a setup of 6 electrodes in use with a symmetrical arrangement. Ions are started from the top of the base electrode (middle of base electrode used as the origo of coordinate system). Using a low learning-factor (0.05) a moderate error-function is generating (shown on Fig. 7).

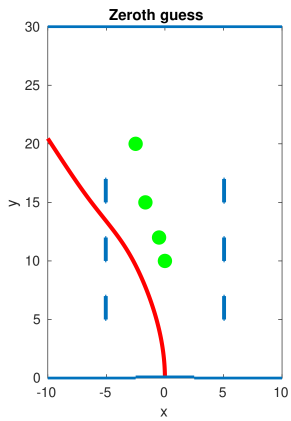

Fig. 8. Trajectory of an ionbeam before after optimization steps. Green disks are control points (1st point not shown, starting point), red line indicates trajectory of ionbeam, electrodes are shown as blue rectangles.

Initial guess of electrode potentials were selected randomly, the resulting trajectory is shown on the Fig. 8. After a few iteration steps the trajectory takes up its best way, show on Fig. 9. Control points (green disks) can have different weights, points that are closer to the end of nozzle have higher weights because they give the main curve of the outflying path. As it can be seen this guess has some big problems and couldn't be realizable.

After optimization the trajectory is shown on Fig. 9. Path of ions follows the predescribed route.

We found that number of electrode-pairs determines the possible control points. If we choose nearly the same number of control points as the number of electrode-pairs then optimization will have a good output in terms

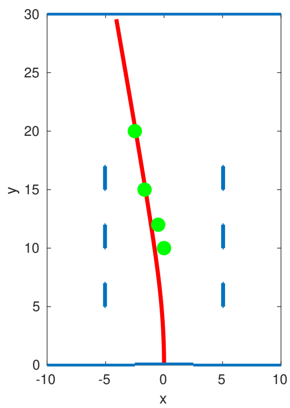

Fig. 9. Trajectory of an ionbeam after optimization steps. Notations are the same as on Fig. 8.

of error defined earlier. Of course not all routes can be realized because the ions have inertia and it doesn't allow arbitrary changes in direction.

Our simulations were performed in a two-dimensional environment that used the symmetry of the rectangular shape nozzle, so control of ionbeams are available only in one direction.

V. Summary

In this paper, we solved the problem of optimizing control electrodes' potential to drive ion-beam on the prescribed path. Optimization is realized using a machine-learning algorithm to be able to realize on special hardware later. It was shown that control of the ion-beam trajectory could be done due to the adjustment of electrodes' potential.

SML is highly sensible to start point of the parameter vector, and human supervision is needed sometimes. Since SML finds only local minima, therefore, perturbing start parameter vector should sometimes be carried out.

In the future, we plan to use special hardware for implementation three-dimensional simulation of the structure to analyze selectivity of the trajectory control.

Solution of this kind of problem can give guidance for the designers how to choose the appropriate number and location of the control electrode to achieve maximum maneuverability of tiny satellites.

The solution's computational cost in case of a three dimensional direct problem is much greater than of this two-dimensional problem therefore this simple guess can be a good initial state of the further (3d) solutions.

References

[1] SMOG-1, Official Site, http://gnd.bme.hu/smog
[2] J. McDowell, List of all known satellites launched, http://planet4589.org/space/log/launchlog.txt
[3] Arianespace, Ariane 5 Users Manual Issue 5 Revision 2, October, 2016.
[4] Wilfried Ley, Klaus Wittmann and Willi Hallmann (ed): Handbook of Space Technology, Wiley, 2009.
[5] Choueiri, Edgar Y (2009). "New dawn of electric rocket". Scientific American. 300 (2): 5865.
[6] Martinez-Sanchez, M., Pollard, J.E. Spacecraft Electric Propulsion An Overview. J. Propul. Power, 14 (5), 688693, 1998.
[7] A. Makara, A. Reichardt and L. Csurgai-Horvath, "Visualization and simulation of ion thrusters possibly usable by small satellites", H-SPACE2020, Budapest, 2020
[8] Jahn, Robert G. (1968). Physics of Electric Propulsion (1st ed.). McGraw Hill Book Company
[9] K. Simonyi, "Theoretische Elektrotechnik", Leipzig : Barth, 1993.
[10] R. Hockney, J. Eastwood, "Computer Simulation Using Particles", IOP, New York, 1989.
[11] Jahn, Robert G. (1968). Physics of Electric Propulsion (1st ed.). McGraw Hill Book Company
[12] Kevin P. Murphy, "Machine Learning A Probabilistic Perspective", 2012, MIT Press
[13] Martin T. Hagan et al., "Neural Network Design" , 2nd ed., E-Book, 2014